Benchmarking of a Three-Dimensional Limit Equilibrium Slope Stability Software

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ABSTRACT
2D slope stability analyses have been performed using various software packages on a routine basis in geotechnical engineering since the 1980s. Many of these 2D packages are now used by geotechnical consultants, often with little regard for the potential differences between the various analysis methods implemented in different software packages. It is difficult to know how to adequately benchmark slope stability software package as the new features are often implemented faster than the related benchmarks are created. Many of the early software packages do not appear to have been adequately tested against a significant number of comprehensive benchmark examples. Recently there has been an emphasis on more rigorous 3D slope stability analysis and a new group of benchmarked models has been developed. This paper summarizes the results of an extensive comparison of benchmark results obtained during the development of the SVSLOPE-3D software package. Issues surrounding the correct solution of benchmark example problems are discussed. The potential applications of this technology are also discussed.

RÉSUMÉ
Les analyses de stabilité de talus 2D ont été effectuées régulièrement dans le domaine du génie géotechnique à l'aide de divers logiciels depuis les années 1980. Un grand nombre de ces paquets 2D sont maintenant utilisés par des consultants en géotechnique, souvent avec peu d'égard pour les différences potentielles entre les différentes méthodes d'analyse mises en œuvre dans différents logiciels. Il y a de la difficulté à établir des exemples de repère suffisant pour les progiciels de stabilité de talus puisque les nouvelles fonctionnalités sont souvent mises en œuvre plus rapidement que les points de référence sont créés dans le monde universitaire. Il semble que la plupart des premiers progiciels n'aient pas été vérifié contre un nombre suffisamment important d'exemples de référence complets. Les dernières décennies ont vu davantage l'accent sur une analyse de stabilité de talus 3D plus théoriquement correcte et donc un nouveau groupe de modèles étalonnés a été développé. Le présent document résume les résultats d'une comparaison approfondie des résultats de référence obtenus au cours du développement du progiciel SVSLOPE-3D. Les questions entourant la solution correcte des exemples de référence sont discutées. Les applications potentielles de cette technologie seront également discutées.

1 INTRODUCTION

Slope stability problems in geotechnical engineering involve the solution of equilibrium equations of force and moment. Equilibrium equations are traditionally satisfied through the use of method of slices techniques or more progressive stress-based methods.

Current industry practice often requests that a 2D slope stability analysis be performed. The 2D analysis assumes that the slope is homogenous with respect to the third dimension and that the slip surface is cylindrical (or composite) in shape. Gitirana et al. 2008 found that analyzing problems in 3 dimensions can lead to differences in the lowest factor of safety, F_s, between 15% and 50%. It is prudent for engineers to give further consideration to 3D analyses. Three-dimensional slope stability modeling can give consideration to more complex and realistic, real-world scenarios. The end result is greater confidence in the slope stability results.

This paper presents some of the extensive three-dimensional benchmarking which has been performed on the SVSLOPE software package. Comparisons are made to literature scenarios, as well as to other software package results.

1.1 Reasons for Verification

"Verification" is generally achieved by solving a series of so-called "benchmark" problems. "Benchmark" problems are problems for which there is either a closed-form solution or for which the solution has become "reasonably certain" as a result of longhand calculations that have been performed. Benchmarks can be drawn from historical failure scenarios, literature studies, as well as through comparisons to other software. Publication of the "benchmark" solutions in research journals or textbooks also lends credibility to the solution. It must be remembered that there is no such thing as a complete software verification for "all" possible problems. Rather, it is an ongoing process that establishes credibility with time.

2 OVERVIEW OF SLOPE STABILITY MODELING

Figure 1 provides an overall classification of slope stability methods of analysis. A distinction is made between analysis methods and searching techniques. In 1977, Fredlund and Krahn classified limit equilibrium methods of slices according to the elements of statical equilibrium that were satisfied when solving for the factor of safety. The classification also took into consideration the assumptions used to render the analysis determinate.

In 1981, Fredlund, Krahn and Pufahl further extended the comparison of slope stability method of slices to include additional methods of slices (Fredlund et al, 1981). Most of the limit equilibrium methods of slices made an assumption regarding the interslice forces (e.g., the interslice force function). Consequently, most of the methods of slices differed in the manner in which the normal force at the base of a slice was calculated. Common to all the methods of slices was the manner in which the factor of safety was defined and the fact that the normal force was computed from statical considerations of one slice of a potential sliding mass.

Figure 1 shows that it is now possible to also take into consideration the search technique associated with the determination of the shape and location of the critical slip surface. The finite element stress analysis method can also be used to determine the normal force at the base of a slice, giving rise to the Enhanced (Kulhawy) Limit method as well as other optimization techniques (e.g., Dynamic Programming).

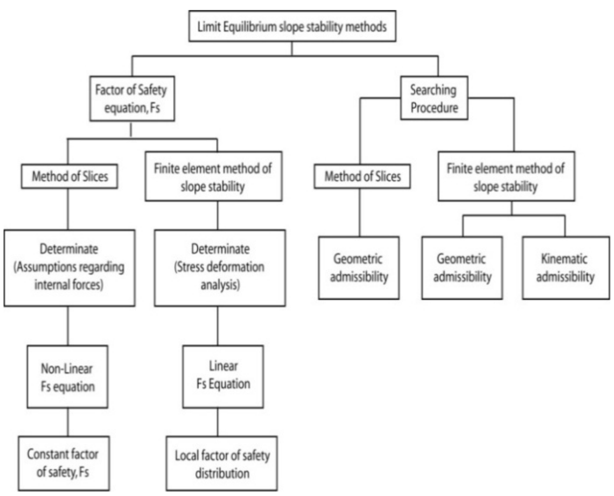

Figure 1. Overall classification of slope stability methods of analysis

2.1 3D Slope Stability

"Methods of slices" becomes the "method of columns", when extending a slope stability analysis into 3 dimensions. When considering a "slice" of a sliding mass, 2-dimensional inter-slice forces are designated. However, when a three-dimensional column is considered, lateral forces also come into effect, creating a much more realistic simulation of the real-world conditions. Three-dimensional analyses require the computation of additional variables. However, the same general slope stability formulations that are commonly performed in 2D (e.g., Bishop, Morgenstern-Price, Janbu, etc.) can be extended into 3D.

The circular sliding surfaces commonly found in 2D analyses can also be extended to an ellipsoidal sliding surface for 3D analyses. Other shapes can also be considered in 3D analyses. For example, multi-planar wedges and fully specified sliding surfaces along known failure zones can be specified. The software also allows other features to be considered such as external loads, supports, bedrock (slip surfaces cannot pass through these zones), tension cracks and discontinuities.

3 BENCHMARKING

In the following sections some of the benchmarks used to test the SVSLOPE 3D solution engine are presented. More than 20 benchmarks have currently been created during the initial testing of the SVSLOPE 3D software. Ten (10) of the created benchmarks are currently published in the SVSLOPE 3D verification manual. This paper highlights some of the benchmarks created during the software testing research program at SoilVision Systems Ltd.

3.1 Kettleman Hills Waste Landfill Failure

This example simulates the actual failure of the Kettleman Hills waste landfill (Seed, Mitchell and Seed, 1990). The slip surface can be modeled using a multi-planar wedge surface. Three wedge planes are associated with 3 different material discontinuities as shown in Table 1.

The geometry and material properties are shown in Table 1 and Figure 2.

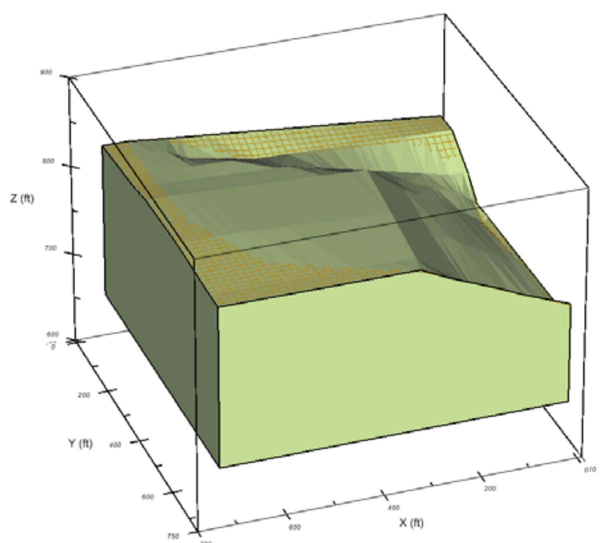

Figure 2. Geometry of the Kettleman Hills Landfill model (2x vertical exaggeration)

Table 1. Material Properties of the Kettleman Hills Landfill model (including discontinuities)

	c (psf)	ϕ (degrees)	γ (lb/ft^3)
Mat1	0	20	110
Dis1	0	8	127
Disc2	0	8.5	127
Dis3	900	0	127

The results are presented in Table 2 and Figure 3. The differences between the software packages are deemed negligible.

Table 2. Results of the Kettleman Hills Landfill model

Method	Factor of Safety		Difference
	CLARA/W	SVSLOPE3D	(%)
Bishop	1.160	1.159	0.057
Janbu Simliplified	1.140	1.145	0.416
Spencer	1.160	1.168	0.655
M-P	1.170	1.164	0.551

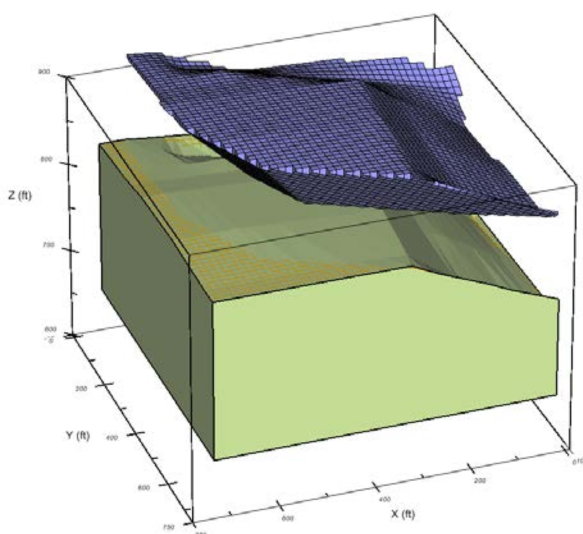

Figure 3. Failure Sliding Surface of Kettleman Hills Landfill model with critical slip surface exploded out of the slope (2x vertical exaggeration)

3.2 Pore-Water Pressures at Discrete Points

This model is used to benchmark the discrete points method to specify the pore water pressures in SVSLOPE 3D. The discrete points (specified in terms of pressure heads) in this model are used to simulate the water table surface in the original Ellipsoidal Toe Submergence model.

The model geometry and material properties are the same as the Ellipsoidal Toe Submergence model with a change of the initial condition from a water surface to discrete points (pressure head). The model geometry is shown in Figure 4 including contouring of the discrete points (in terms of pressure head).

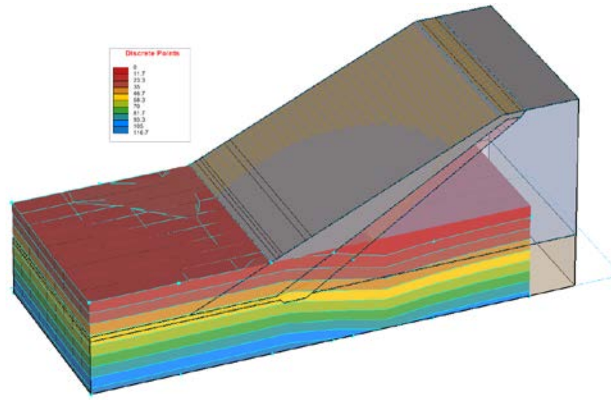

Figure 4 Geometry pore-water pressurediscrete points model with discrete points contoured

The Results are shown in Table 3. The results from CLARA/W are based on the pore-water pressure dataset with a watertable surface. The differences are deemed to be negligible.

Table 3. Results of the PWP discrete points model

Method	Factor of Safety			Difference
	CLARA/W	SVSLOPE 3D		(%)
		Moment	Force	
Bishop	1.30	1.296		0.339
Janbu Simplified	1.23		1.229	0.090
Spencer	1.26	1.239	1.239	1.652

3.3 Tension Cracks in 3D

The model geometry and material properties are the same as the previously published Ellipsoidal Toe Submergence model with the addition of the tension crack information. The tension crack is specified using a x-coordinate equal to 300. Eighty percent of the tension crack is assumed to be filled with water.

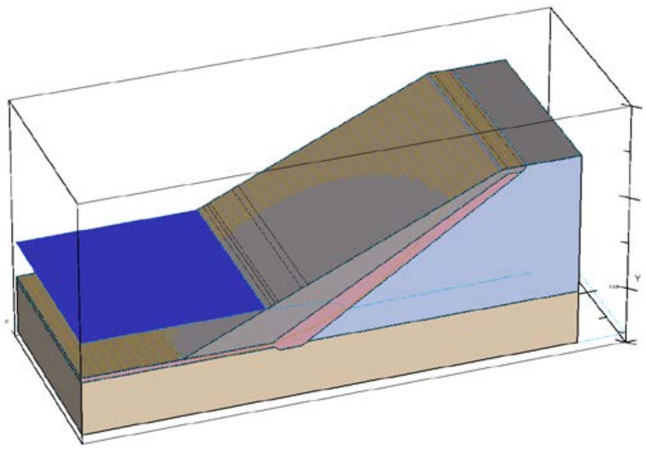

Figure 5 Geometry of the Ellipsoidal Toe Submergence model with the water surface

The Results are shown in Table 4 and Figure 6.

Table 4. Results of the Tension Crack model

Method	Factor of Safety			Difference
	CLARA/W	SVSLOPE 3D		(%)
		Moment	Force	
Bishop	1.26	1.236		1.879
Janbu Simplified	1.19		1.170	1.708
Spencer	-	1.247	1.246	-

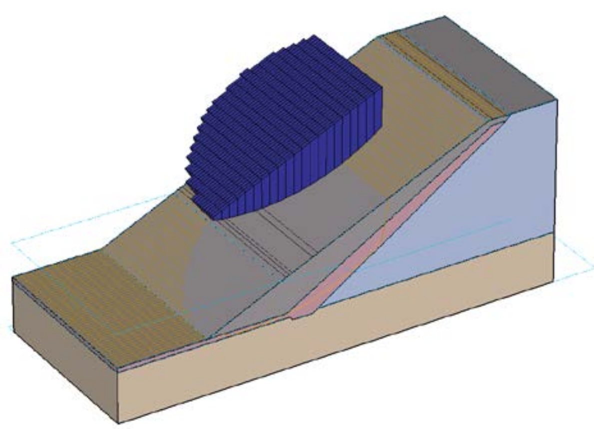

Figure 6 Result of the Tension Crack model with critical slip surface exploded out of the slope

3.4 General Sliding Surface

This example demonstrates the use of general sliding surface. In the original “Example 6” in CLARA/W the model used a Hoek-Brown strength model for the shale bedrock layer material, since there is a different implementation of the Hoek-Brown model in CLARA/W and SVSLOPE, the bedrock material strength model is changed to a Mohr-Coulomb in both the CLARA/W and SVSLOPE 3D software packages for the convenience of comparison.

The pore-water pressures are specified with a water surface (grid data). The geometry and material properties are shown in Figure 7 and Table 5.

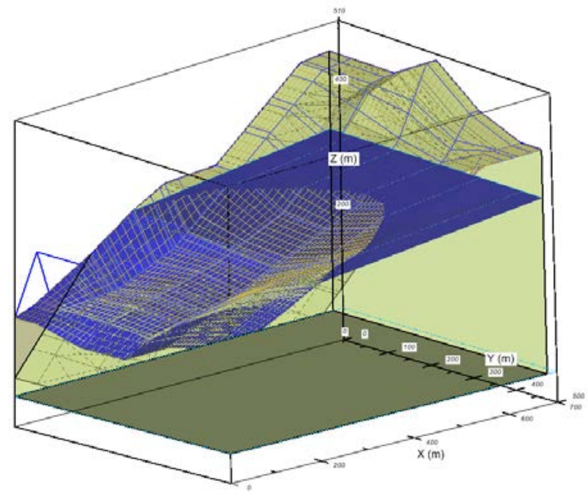

Figure 7 Geometry of the General sliding surface model

Table 5. Material Properties of General sliding surface model

	c (kN/m^2)	ϕ (degrees)	γ (kN/m^3)
Glacial Till	0	35	22
Rock	100	45	26

The results are shown in Table 6 and Figure 8. The slight differences between the software packages are considered reasonable.

Table 6 Results of General sliding surface example

Method	Factor of Safety		Difference
	CLARA/W	SVSLOPE3D	(%)
Bishop	2.22	2.227	0.334
Janbu Simplified	2.170	2.180	0.457
Spencer	2.230	2.184	2.070

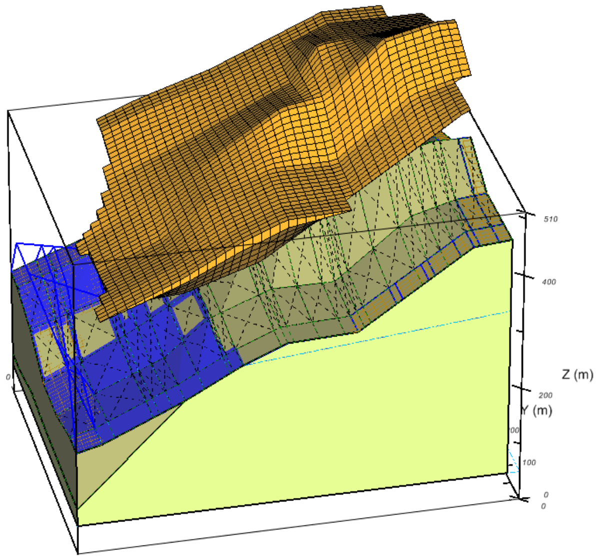

Figure 8 General sliding surface using the extended Bishop Method with critical slip surface exploded out of the slope

3.5 A Simple 3D Slope In Clay

This model represents a three-dimensional slope stability problem in clay. The model involves a spherical failure surface in clay and is often used in the literature as a benchmark example against which numerical models have been validated (Hungr 1989, Silverstri 2006).

The slope geometry and material properties utilized for this model are presented in Figure 9 and Table 7. The requirements for this problem are the factor of safety and its comparison to its closed-form solution.

Table 7. Material Properties of the Simple Slope model

c (kN/m^2)	ϕ (degrees)	γ (kN/m^3)
0.1	0	1

The fully specified ellipsoid (spherical) slip surface is utilized in the analysis to make a comparison with published results. The sphere radius is 1.0 and its center is located at (4.780, 5, 7.960).

There are 42 rows and 42 columns used in the analysis which results in a total of 872 active columns. The factor of safety for the extended Bishop's method is 1.398. A summary of the factors of safety for this benchmark example is presented in Table 8.

Table 8. A summary of factors of safety for the simple 3D slope in clay

Method	Factor of Safety	Difference (%)
Closed-Form Solution (Hungr et al. 1989)	1.402	1.816
Closed-Form Solution (Silverstri 2005)	1.377	-
CLARA Solution 42x42 (Hungr et al. 1989)	1.400	1.643
3D-SLOPE solution (Lam, et al. 1993)	1.402	1.816
SVSLOPE 3D	1.398	1.525

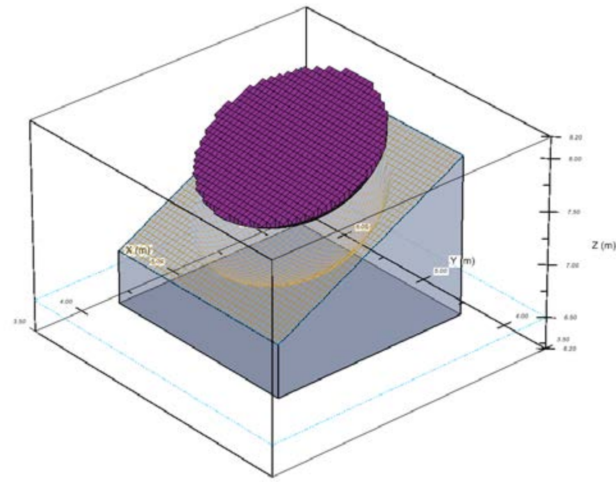

Figure 9. Geometry and results of the Simple 3D Slope model in Clay

3.6 A Model Compared To Variational Approach

Leshchinski et al. (1985) proposed an analytical solution for sliding surfaces with logarithmic spirals. It satisfies all equilibrium conditions. Lateral equilibrium is met by symmetry. The slope geometry and material properties that are in use for this model are presented in Figure 11. Hungr et al. (1989) presented the geometry in detail.

The fully specified ellipsoid slip surface with aspect ratio = 0.66 is used in the analysis to make a comparison with published results. The center of the ellipsoid is located at (-0.67, 0, 1.737).

The result is shown in Figure 10. A summary of the factors of safety for this benchmark example is presented in Table 9. The results match CLARA/W with a difference of less than 1.6%.

Table 9. A summary of factors of safety for the Hungr Leshchinski 3D model

Method	Factor of Safety	Difference (%)
Analytical Solution (Leshchinski et al. 1985)	1.25	-
CLARA/W Solution (Hungr et al. 1989)	1.23	1.6
SVSLOPE 3D	1.245	0.4

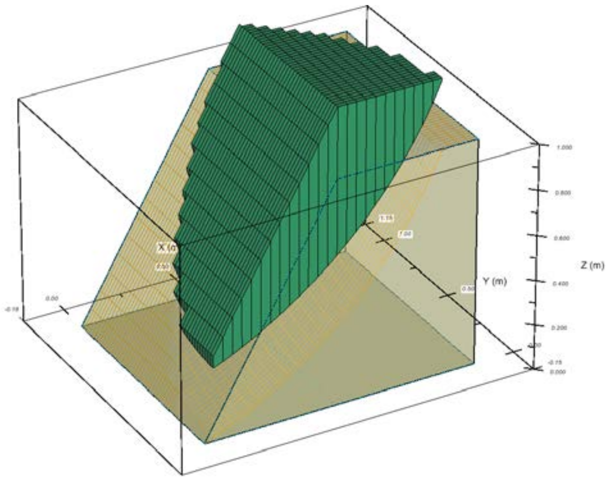

Figure 10. Result of the Hungr-Leshchinski 3D model

3.7 Embankment Corner

This model represents an embankment corner. The grid and tangent search method is utilized to identify the critical slip surface.

There is no pore-water pressure input for this problem. The geometry and material properties are shown in Figure 11 and Table 10.

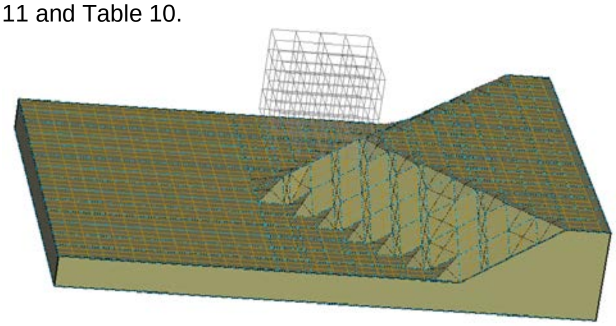

Figure 11. Geometry of Embankment Corner model

Table 10. Material Properties of the CLARA_example3 model

	c (kN/m^2)	ϕ (degrees)	γ (kN/m^3)
Mat1	10	22	20

The following results are obtained using the grid and tangent search technique and are shown in Table 11 and Figure 12. A maximum difference of 2.715% was noted which is reasonable.

Table 11. Results of the Embankment Corner

Method	Factor of Safety		Difference
	CLARA/W	SVSLOPE3D	(%)
Bishop	1.824	1.836	0.642
Janbu Simplified	1.560	1.578	1.165
Spencer	1.784	1.832	2.715
M-P	1.830	1.822	0.434

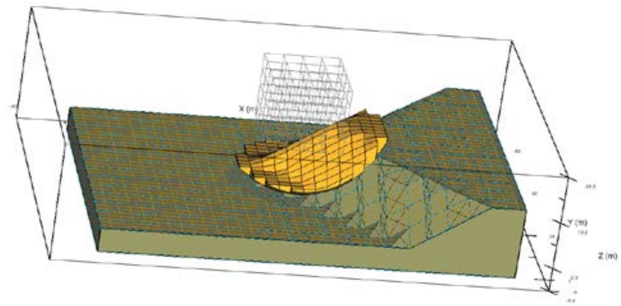

Figure 12. Result of Embankment Corner model with Bishop Simplified method

3.8 Bedrock Layer Consideration

This is a simple symmetrical slope problem therefore only half is analyzed. An ellipsoidal sliding surface is utilized. The lower material layer is bedrock and the sliding surface is not allowed to penetrate the bedrock. The ellipsoidal sliding surface will be cut off when passing through the bedrock layer. CLARA/W's Spencer method does not converge in this model.

The geometry and material properties are presented in Figure 13 and Table 12.

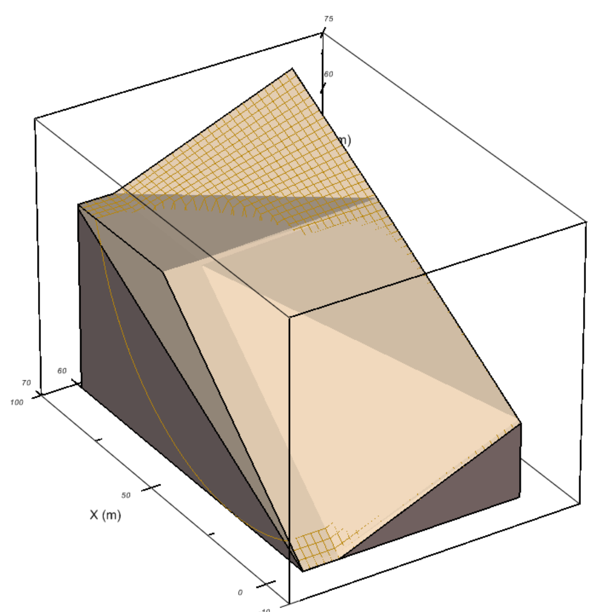

Figure 13. Ellipsoidal Sliding Surface

Table 12 Material Properties of Bedrock model

	c (kN/m^2)	□ (degrees)	γ (kN/m^3)
Material2	15	25	20

The results are shown in Figure 14 and Table 13. The differences between the software packages are considered negligible.

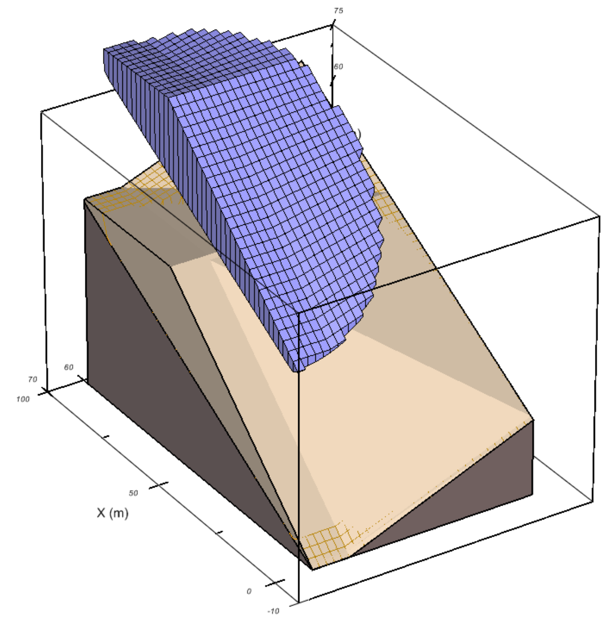

Figure 14. Failure sliding surface of the Bedrock model

Table 13 Results of the Bedrock model

Method	Factor of Safety		Difference
	CLARA/W	SVSLOPE3D	(%)
Bishop	1.20	1.238	3.187
Janbu Simliplified	1.17	1.210	3.439
M-P	1.19	1.227	3.116

3.9 Multiple Piezometric Surfaces

There are six layers in this model. Each layer is associated with a different piezometric surface in order to simulate the condition of upward seepage.

A fully specified Ellipsoidal sliding surface is used in this analysis, the geometry and material properties are shown in Table 14 and Figure 15.

Table 14. Material Properties of the multi_piezo_surfaces model

	c (kN/m^2)	□ (degrees)	γ (kN/m^3)
Clayer1	20	18	19

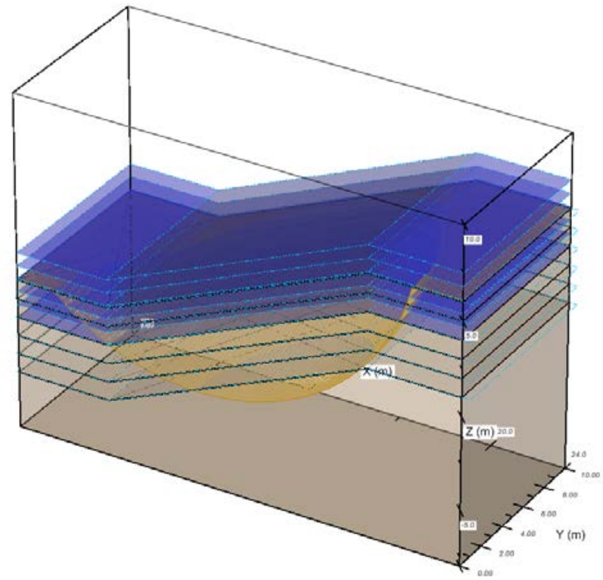

Figure 15. Geometry of the multi_piezo_surfaces model

The results are shown in Figure 16 and Table 15. Negligible differences between the software packages are noted.

Table 15. Results of the multi_piezo_surfaces model comparison

Method	Factor of Safety		Difference
	CLARA/W	SVSLOPE3D	(%)
Bishop	2.15	2.163	0.616
Janbu Simliplified	1.93	1.939	0.457
Spencer	2.15	2.145	0.249
M-P	2.16	2.137	1.059

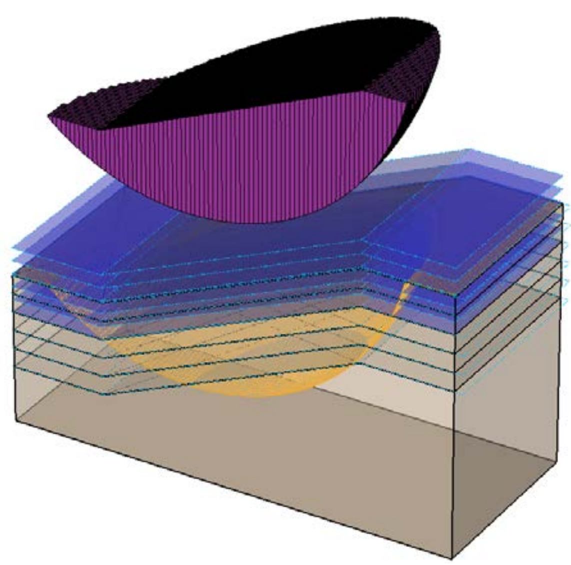

Figure 16. Failure sliding mass of the multi_piezo_surfaces model

4 POTENTIAL APPLICATIONS

Three-dimensional slope stability analyses are relevant with respect to most natural landslides. Most natural geometries are too complex to be completely modeled using a two-dimensional analysis. In general, there is benefit in also performing a more rigorous three-dimensional slope stability analysis. Some problems that are inherently three-dimensional are:

- **Levee systems:** Levee systems can extend over many miles. The transportation and placement of earthfill is generally quite costly. Most existing designs are based on a 2D analysis; however, there may be cost savings that result from performing a three-dimensional analysis.
- **Pit slope stability:** The highly 3D nature of mining pits means that some parts of the geometry should be analyzed using a 3D analysis. The limit equilibrium method is well-suited for the highly irregular geometry often encountered in pit geometries.
- **Earth storage:** Often it is necessary in the mining industry to store borrow materialsfor long periods of time (e.g., top soil used for reclamation). By stacking the material higher and making the slopes steeper the area required for storage can be reduced. A 3D analysis allows the calculation of more realistic factors of safety and allows the opportunity to optimize the design of the storage facilities.
- **Tailings and mine rock piles:** The land requirement for the storage of tailings and mine rock piles can be costly. An optimized design performed using a 3D analysis allows for the storage of more tailings per unit area.
- **Heap leach analysis:** Larger amounts of ore can be extracted on a heap leach pile when steep side slopes can be used. However, the increased side slope steepness results in a condition closer to failure. A 3D analysis can be used to more realistically represent the geometry of the heap leach pile. It is important to accurately balance the slope angle design and the ore recovery.

5 CONCLUSIONS

Comprehensive verification of engineering software is critical to the application of new methodologies and the establishment of confidence related to various new methodologies. Development of a new software package has required a significant benchmarking effort. This paper presents several benchmark examples that have been created to validate the use of the three-dimensional, slope stability software in geotechnical engineering. Other verification examples are published in the SVSlope verification manual available on the SoilVision System Ltd. website.

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