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Comparison of Measured and PTF Predictions of SWCCs for Loess Soils in China

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Abstract There are significant advantages in using indirect pedo-transfer functions, (PTFs) for the estimation of unsaturated soil properties. The pedo-transfer functions can be used for the estimation of the soil–water characteristic curve (SWCC) which in turn is used for the estimation of other unsaturated soil properties. The accuracy of the indirect pedo-transfer function method for the estimation of the SWCC depends on the PTF and the equation used to best-fit the particle-size distribution (PSD) data. The objectives of this study are to: (1) evaluate the performance of the Fredlund et al. (Can Geotech J 37:817–827, 2000) equation for best-fitting the particle-size distribution, (PSD) data, and, (2) compare the predictions made by two of the commonly used PTFs; namely, Arya and Paris (Soil Sci Soc Am J 45:1023–1030, 1981) and Fredlund et al. (Can Geotech J 39:1103–1117, 2002), for estimating the SWCC from the PSD. The authors used 258 measured PSDs and SWCC datasets from the Loess Plateau, China, for this study. The dataset consisted of 187 silt–loam soils, 41 loam soils, 11 silt–clay–loam soils, 10 sand–loam soils, 6 silt–clay soils, and 3 loam–sand soils. The SWCC and PSD datasets were measured using a Pressure Plate apparatus and the pipette method, respectively. The comparison between the estimated and measured particle-size distribution curves showed that the Fredlund et al. (Can Geotech J 37:817–827, 2000) equation closely prepresented the PSD for all soils in the Loess Plateau, with a lower root mean square error (RMSE) of 0.869%. The comparison between the estimated and measured water contents at the same suction showed that the Fredlund et al. (Can Geotech J 39:1103–1117, 2002) PTF performed somewhat better than the Arya and Paris (Soil Sci Soc Am J 45:1023–1030, 1981) function. The Fredlund et al. method had RMSE value of 0.039 $cm^3\ cm^{-3}$ as opposed to 0.046 $cm^3\ cm^{-3}$ for the Arya and Paris (Soil Sci Soc Am J 45:1023–1030, 1981) method. The Fredlund et al. (Can Geotech J 39:1103–1117, 2002) PTF produced the closest predictions for sand–loam, loam–sand, and loam soils, with a lower RMSE for gravimetric water content ranging from 0.006 to 0.036 $cm^3\ cm^{-3}$. There were consistent over-estimations observed for silt–loam, silt–clay–loam, and slit–clay soils with RMSE values for gravimetric water content ranging from 0.037 to 0.043 $cm^3\ cm^{-3}$. The measured and estimated air-entry values were closest when using the Fredlund et al. (Can Geotech

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J 39:1103–1117, 2002) PTF. The measured and estimated maximum slopes on the SWCC were closest when using the Arya and Paris (Soil Sci Soc Am J 45:1023–1030, 1981) PTF.

Keywords Soil–water characteristic curve · SWCC · Particle-size distribution · PSD · Pedo-transfer functions · PTF · Unsaturated soil property functions

1 Introduction

The description and mathematical simulation of water movement through unsaturated soils requires information on water transmission and water storage properties. These properties are commonly known as hydraulic conductivity, (or coefficient of permeability), and water storage modulus. The field and laboratory determination of hydraulic conductivity and water storage for unsaturated soils are laborious and costly (van Genuchten and Leij 1992). This has led to the development and use of indirect methods which are known as Pedo-Transfer Functions (PTF). Over the past two decades, a variety of PTFs have been developed based on soil particle- size distribution, PSD, parameters and other basic geotechnical properties. The capillary theory has been used to relate void space between particles to their ability to retain water (Arya and Paris 1981).

Pedo-transfer functions can be categorized into two groups based upon the respective estimation techniques. The first group of PTFs uses statistical estimates of soil properties to describe the SWCC. The soil properties are particle-size and volume-mass properties (Gupta and Larson 1979; Rawls and Brakensiek 1989; Vereecken et al. 1989; Tyler and Wheatcraft 1990; Scheinost et al. 1997; Schaap and Leij 1998). The second group of PTFs utilizes a physico-empirical approach that converts particle-size distributions into pore-size distributions. The pore-size distributions are then used to develop a SWCC (Arya and Paris 1981; Aubertin et al. 2003). Several authors have evaluated the performance and suitability of different Pedo-Transfer Functions for estimating hydraulic properties (Espino et al. 1995; Sobieraj et al. 2001). In general, it has been found that the performance of PTDs is largely dependant on the dataset used for the calibration of the model (Schaap and Leij 1998). Inaccurate predictions often occur when predictions are made for soils that are outside the range of soils that were used for calibrating the PTDs (Cornelis et al. 2001; Hodnett and Tomasella 2002). The two previously mentioned approaches have encountered some difficulties in generating a reasonable SWCC along the entire range of soil suctions (Fredlund et al. 1997). For example, the predicted SWCC often falls off to zero volumetric water content before the experimental data are completely desaturated. Fredlund et al. (2002) developed a method to predict entire SWCC using particle-size distribution curves. A packing factor was incorporated to represent soil porosity (or the packing between particles of individual grain sizes).

There are few independent measurements required in the verification of PTFs; namely, the particle-size distribution (PSD) and the soil–water characteristic curve (SWCC). There is need to establish increased confidence in the relationship between the PTF and the SWCC. It is suggested that one way to improve accuracy when using the PTF methodology is to use a mathematical equations to best-fit the PSD data sets. This equation can then be used to estimate the PTF. Wagner and Ding (1994) reviewed previous research studies related to the best-fit of PSDs, and found that several lognormal distributions were capable of fitting the central portion of the particle-size distribution. However, providing a meaningful representation of PSD data at the extremities proved to be difficult when using a lognormal distribution. Hwang et al. (2002) compared the capability of seven PSD models with different underlying assumptions. The PSD models were used to best-fit experimental PSD data on 1,387 soils in a Korean soil database. It was found that the three-parameter Fredlund and Xing (1994) equation performed best. The Fredlund and Xing (1994) equation, which had been used to best-fit SWCC data, provides a flexible and continuous function that can be best-fit using a nonlinear regression analysis of three fitting parameters. Fredlund et al. (2000, 2002) modified the Fredlund and Xing (1994) SWCC equation to permit the fitting of a continuous function, even in the extremes of PSD curves.

The accuracy of the indirect method for the SWCC depends on the estimated PTF and the equation used to best-fit the PSD data. The objectives of this present study are to: (1) evaluate the performance of the Fredlund et al. (2000) equation for best-fitting PSD curves, and, (2) compare the predictions made by two of the commonly used PTFs; namely, Arya and Paris (1981) and Fredlund et al. (2002), for estimating the SWCC from the PSD. A dataset consisting of 258 soil samples from the Loess Plateau, China consisted of PSD and SWCC measurements. The dataset included 187 silt–loam soils, 41 loam soils, 11 silt–clay–loam soils, 10 sand–loam soils, 6 silt–clay soils, and 3 loam–sand soils.

2 Materials and Methods

2.1 Fredlund et al. (2000) Equation for Estimating PSD

The PSD model selected for this study is the Fredlund et al. (2000) unimodal and bimodal model. The bimodal PSD model showed advantages over the unimodal model in some cases. However, the total number of fitting parameters is doubled when using the bimodal model. More detailed experimental data are required in order to justify the usage of the bimodal PSD equation. The number of experimental PSD data points for each soil used in this study was only six. Therefore, the parameters for the bimodal PSD model cannot be justified and only the unimodal PSD model was used.

The unimodal model to represent the grain-size distribution given by Fredlund et al. (2000) is as follow:

$$P_d(d) = \frac{1}{\ln\left[e + \left(\frac{a_{\mathrm{gr}}}{d}\right)^{n_{\mathrm{gr}}}\right]^{m_{\mathrm{gr}}}} \left\{ 1 - \left[\frac{\ln\left(1 + \frac{d_{\mathrm{r}}}{d}\right)}{\ln\left(1 + \frac{d_{\mathrm{r}}}{d_{\mathrm{m}}}\right)} \right]^7 \right\} \tag{1}$$

where a_{gr} is a parameter related to the initial breaking point (closest to the largest size particles) on the particle-size curve; n_{gr} is a parameter related to the steepest slope of the curve; m_{gr} is a parameter related to the shape of the fines portion of the curve; d_{r} is a parameter related to the amount of fines in a soil; d is the diameter of any particle size under consideration; and d_{m} is the diameter of the minimum allowable size particle.

2.2 Pedo-Transfer Functions

There are a variety of PTFs in the literature by which the SWCC can be estimated from a PSD. However, considering the available soil property data, the theoretical basis for the analysis, and the ability to correctly predict the SWCC for different soil types, only the Arya and Paris (1981) and Fredlund et al. (2002) PTFs were selected to estimate SWCCs for all soils in the data base.

2.2.1 Arya and Paris (1981) PTF

The Arya and Paris (1981) PTF used the capillary theory to convert the pore radius, r_i, to equivalent pressure head, h_i:

$$h_i = \frac{2\sigma\cos(\beta)}{\rho\, g\, r_i} \tag{2}$$

where σ is the surface tension at the air–water interface, ρ is the density of water, g is the acceleration due to gravity, and β is the contact angle. The β value was assumed to be equal to zero degrees and it was also assumed that it was the drying SWCC that was estimated.

The pore radius was related to the particle radius, R_i, by

$$r_i = 0.816 R_i \sqrt{e n_i^{(1-\alpha_i)}} \tag{3}$$

where α is a scaling parameter, n_i is the number of spherical particles, and e is the void ratio. The volumetric water content θ_i is obtained by summing the water-filled pore volumes according to,

$$\theta_i = (\phi S_w) \sum_{j=1}^{j=i} w_j; \quad i = 1, 2, \ldots, n \tag{4}$$

where ϕ is the total porosity, S_w is the ratio of measured saturated water content to theoretical porosity, and w_i is the fraction solid mass corresponding to particle radius, R_i.

2.2.2 Fredlund et al. (2002) PTF

The Fredlund et al. (2002) PTF for estimating the SWCC assumes that a soil is composed of a series of uniform, homogeneous particle sizes, each leading to a unique SWCC. The general shape of the SWCC for pure sand, pure silt, and pure clay was assumed to be known. Using a best-fit analysis for the Fredlund et al. (2000) equation, three parameters were computed for each soil type. These parameters are assumed to be associated with a dominant particle size on the grain-size plot. It is hypothesized that as a soil tends towards being uniform in size, the values of the fitting parameters show a trend towards a particular value. The fitting parameters for particle sizes falling between pure clays, pure silts, and pure sands are approximated. The particle-size distribution curve can be divided into small divisions with uniform soil particles. The analysis starts from the smallest particle sizes. A packing porosity is estimated for each soil division. The divisional SWCCs are then summed starting with the smallest particle sizes and continuing until the volume of the pore spaces are equal to that of the entire heterogeneous soil. The result is a theoretically estimated SWCC (Fredlund et al. 1997).

2.3 Site Description

All samples comprising the soil data base for this study were taken from the Loess Plateau in China (Fig. 1). The Loess Plateau is located in the upper and middle reaches of the Yellow River and covers a total area of 62.85 × 10^4 km^2 (100° 54′–114° 33′ E and 33° 43′–41° 16′ N) and has an elevation of 1,200–1,600 m above sea level. The loess cover has a thickness ranging from 30 to 180 m (Zhu et al. 1983). The region where the samples were taken is a transitional zone between the southeastern humid monsoon climate and the northwestern continental dry climate. When moving from the southeast to the northwest, the soil texture changes from loam–clay to sand–loam soils according to USA soil classification system (Hillel 1980). Silt–loam soils cover about 90% in of the Loess Plateau. The silt content ranges from 60 to 75% for most soils.

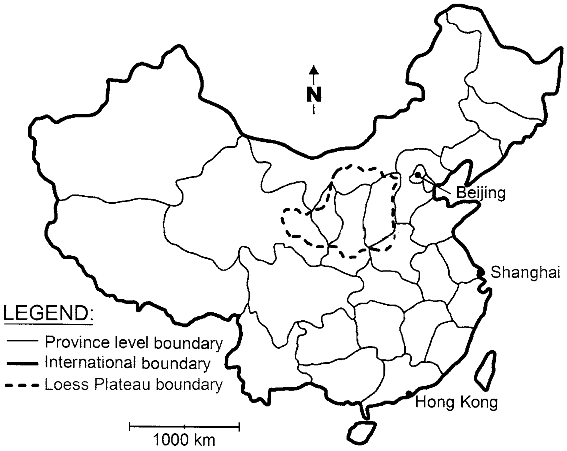


Fig. 1 Location of the study area

2.4 Soil Sampling Strategy

The 258 samples were taken from a depth of 30–60 cm below ground surface in the woodland, grassland, and farmland in 86 counties of the Loess Plateau. Samples were collected for the measurement of soil bulk densities and soil–water characteristic curves. The undistributed samples were collected in duplicate using a 100 cm^3 coring tool from two test pits dug at each site. Particle size analyses were also performed.

Soil–water characteristic curves were measured for the drying curve of each soil using the Pressure Plate method (Smith and Mullins 1991) in which the water content was measured following the application of the following matric suctions; 10, 20, 40, 60, 80, 100, 400, 600, 800, 1,000, 1,200, and 1,500 kPa (i.e., desorption curve). The water content corresponding to each applied pressure was measured once equilibrium conditions were achieved.

Particle-size distribution curves were measured for the following fractions; namely, ≤0.001, 0.001–0.005, 0.005–0.01, 0.01–0.05, 0.05–0.25, and 0.25–1.0 mm using the pipette method (Gee and Bauder 1986). The measured soil bulk densities and particle-size distributions were input to the SoilVision software for calculating the PSD curve and estimating the SWCC. Table 1 shows the soil types along with mean and standard deviation values for the percent passing at various particle sizes.

2.5 Statistical Analysis

Several statistical parameters were used to quantify the difference between the measured and estimated values of percent passing. The same procedure was applied to both the particle-size distribution curve and the SWCC. The mean difference, MD, the root mean square error, RMSE, the intercept, slope and R^2 results of a linear regression are presented. The MD and RMSE are defined as:

Table 1 The soil types and their mean percent passing for different particle sizes (mm)

Soil type	No.		Percent passing, %					
			≤0.001	≤0.005	≤0.01	≤0.05	≤0.25	≤1.0
Silt–loam	187	Mean	15.2	25.6	34.4	83.3	99.5	100.0
		SD[a]	3.6	6.0	7.5	7.3	0.9	0.0
Loam	41	Mean	13.0	23.6	31.7	66.2	91.5	100.0
		SD	3.7	6.1	7.4	5.7	5.1	0.0
Silt–clay–loam	11	Mean	29.2	43.6	54.3	91.0	99.6	100.0
		SD	5.0	6.4	8.7	14.8	2.0	0.0
Sand–loam	10	Mean	8.5	13.1	16.3	39.7	88.5	100.0
		SD	3.5	5.0	5.7	8.3	11.5	0.0
Silt–clay	6	Mean	30.8	44.9	55.6	90.7	99.5	100.0
		SD	4.1	4.3	4.9	3.1	0.4	0.0
Loam–sand	3	Mean	6.7	8.9	10.4	18.0	67.8	100.0
		SD	2.3	1.6	1.2	3.2	9.0	0.0

[a] SD means standard deviation

$$\mathrm{MD} = \frac{\sum_{i=1}^{N}(M_i - E_i)}{N} \tag{5}$$

$$\mathrm{RMSE} = \left[\frac{1}{N}\sum\nolimits_{i=1}^{N}(E_i - M_i)^2\right]^{1/2} \tag{6}$$

where E_i and M_i are predicted and measured values for the ith observation, and N is the total number of measurements.

3 Results and Discussions

3.1 Prediction of Particle Size Distribution

Figure 2 shows the measured and best-fit distributions for selected particle-size distributions. Table 2 contains the RMSEs and the wellness-of-fit values for each soil type. The calculated particle-size distributions from the Fredlund et al. (2000) equation are in good agreement with the measured data. Values of RMSE for the six soil types range from 0.599 to 1.400%. The R^2 values for the six soils ranged from 0.9961 to 0.9996, and the R^2 values on the slopes ranged from 0.993 to 0.999 (Table 2). All statistical results show that the Fredlund et al. (2000) equation can accurately represent the particle-size distributions for all soil types in the Loess Plateau. The differences in RMSE and regression coefficients between the measured and calculated values for the various soil types might result from the different number of observations in each soil category. Amongst the six soil types, the largest number of observations was 187 × 6 points, and the smallest one of 3 × 6 points.

3.2 Effect of Model Parameter on Prediction of SWCC

The scaling parameter α is a key variable when using the Arya and Paris (1981) PTF to estimate the SWCC. Arya and Paris (1981) estimated pore lengths for various fractions of the particle-size distribution curve by summing the diameters of spherical particles in a particular size fraction. Pore lengths based on spherical particles were scaled to natural pore lengths using a scaling parameter, α, which was found to have an average value of 1.38. Later investigations by Arya et al. (1982) showed that the average α value varied from one soil textural classes to another. Correspondently, the α value ranged from 1.1 for fine-textured soils to 2.5 for coarse-textured soils. Arya et al. (1999) carried out further investigations and found that the α value was not constant but decreased with increasing particle diameters. An empirical formulation was developed for the estimation of suitable α values. The empirical formulation is:

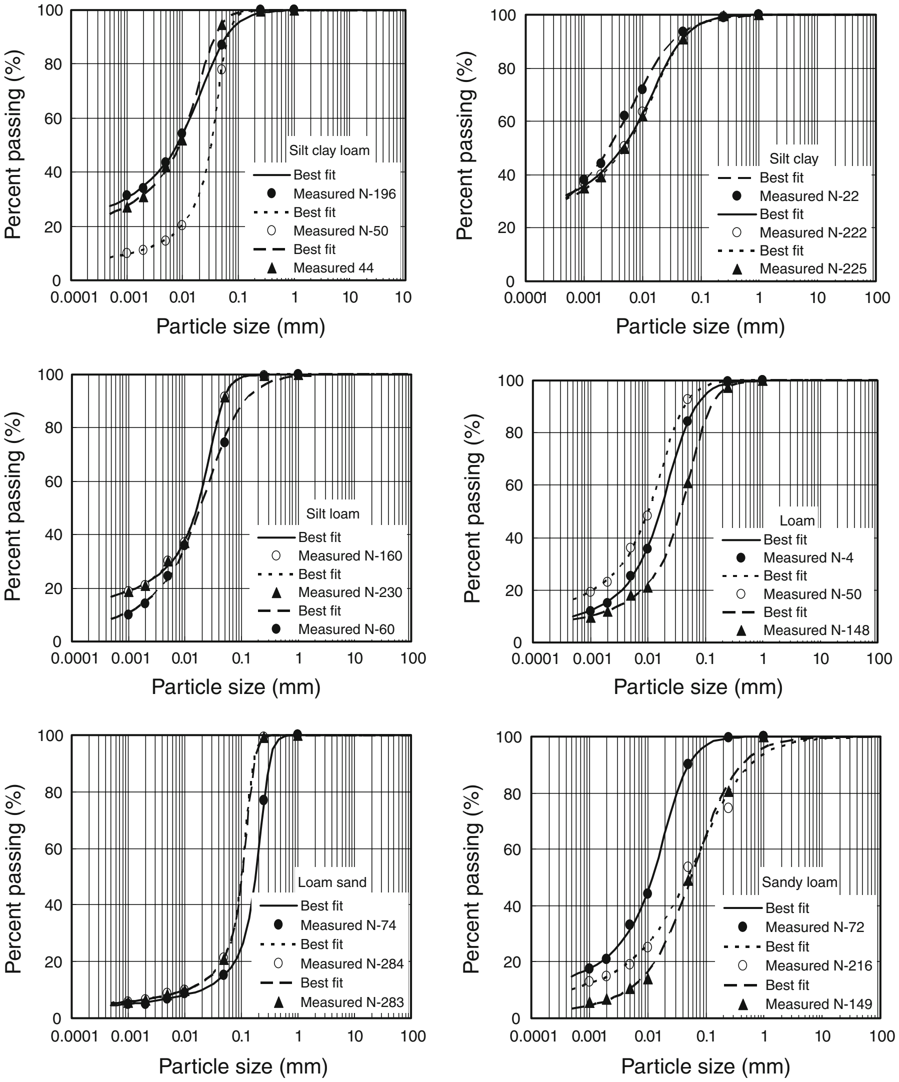


Fig. 2 Prediction of particle-size distribution using the Fredlund et al. (2000) equation for selected soils

$$\alpha_i = \frac{a + b\log\left(w_i/R_i^3\right)}{\log n_i} \qquad (7)$$

where a and b are parameters (Table 3); R is particle radius; w_i is the solid mass fraction corresponding to particle radius; n_i is the number of spherical particles.

The effect of a constant value of α versus a continuous function (i.e., Eq. 7), on the prediction of the SWCCs was studied by Arya et al. (1999). The final results showed that considering α as a continuous function could not significantly improve the predictive accuracy of SWCCs; 23 soils were tested (Arya et al. 1999). The effect of having α as a constant or a continuous function on the prediction of the SWCC prediction was also studied for three soil textures; namely, silt–loam soil, loam soil, and sand–loam soil. The measured and predicted SWCC for three soils are presented in Fig. 3. For loam soil, a constant α and a continuous function for α resulted in similar prediction of SWCC. For sand–loam and silt–loam soils, however, there was a larger error that occurred in the predicted SWCCs when α was considered as a continuous function. Consequently, in this study, the α variable was assumed as constant for each texture, and the selected values are shown in Table 3.

Table 2 Comparison of the calculated and measured percent passing of particle sizes for all soil types when using the Fredlund et al. (2000) PSD equation

Soil type	No.	Linear regression			RMSE (%)
		Interception	Slope	R^2	
Silt–loam	187 × 6	0.078	0.998	0.9996	0.699
Loam	41 × 6	0.285	0.994	0.9986	1.320
Silt–clay–loam	11 × 6	0.129	0.998	0.9996	0.599
Sand–loam	10 × 6	0.146	0.998	0.9990	1.130
Silt–clay	6 × 6	0.265	0.999	0.9961	1.400
Loam–sand	3 × 6	0.189	0.993	0.9995	0.827
All	1,548	0.123	0.997	0.9994	0.869

Table 3 Parameter values of *a* and *b* and α values proposed by Arya et al. (1999) and Arya and Paris (1981)

Texture	*a*	*b*	α value
Sand	−2.478	1.490	1.459
Sand–loam (loam–sand)	−3.398	1.773	1.285
Loam	−1.681	1.395	1.375
Silt–loam (silt–clay–loam)	−2.480	1.353	1.150
Clay (silt–clay)	−2.600	1.305	1.160

Arya and Paris (1981) did not give α values for all soil types. In this study, the α values of three soil types in parenthesis were approximate

When using the Fredlund et al. (2002) PFT to predict the SWCC, the grain-size distribution curve was divided into *n* fractions of uniformly sized particles. It is possible that the summation of the pore volumes for individual particle-size fractions may be greater than the overall porosity for the combined soil fractions. Therefore, a packing factor, *P*, was assumed for each fraction of soil particles. Ideally, the packing factor should be a function of the particle sizes, but in the Fredlund et al. (2002) PFT, it was assumed to be a constant for all particle sizes. The optimal value for the packing factor for each group of particle sizes was estimated by fitting the predicted SWCC with measured values.

Figure 4 shows the effect of different packing factors, *P*, on the predicted SWCCs for three selected soils. The results show that the packing factor does not always affect the SWCC estimation in the same way. The effect of different *P* values on the prediction of the SWCC varies from one soil type to another. For a silt–loam soil with an increasing *P* value, the predicted gravimetric water contents for all suction ranges decreases significantly. For loam and sand–loam soils, the predicted gravimetric water contents do not show as significant a change as for silt–loam soils. For the sand–loam soil, an increasing *P* value does not affect the predicted water content in the range from zero to 1 kPa, and the predicted water contents significantly decrease beyond a suction of 1 kPa. Therefore, the packed porosity has a significant influence on the predicted SWCC. It is suggested that further research should be undertaken on the role of the packing factor on the prediction of the SWCC.

3.3 The PTF Best-Fit for SWCC

Comparisons between the measured and predicted SWCC when using a constant α with the Arya and Paris (1981) PTF and the best-fit packing factor in Fredlund et al. (2002) PTF are shown in Fig. 5 for six soils. There appears to be greater difficulty in estimating the SWCC for silt–clay soils, silt–clay–loam soils, and silt–loam

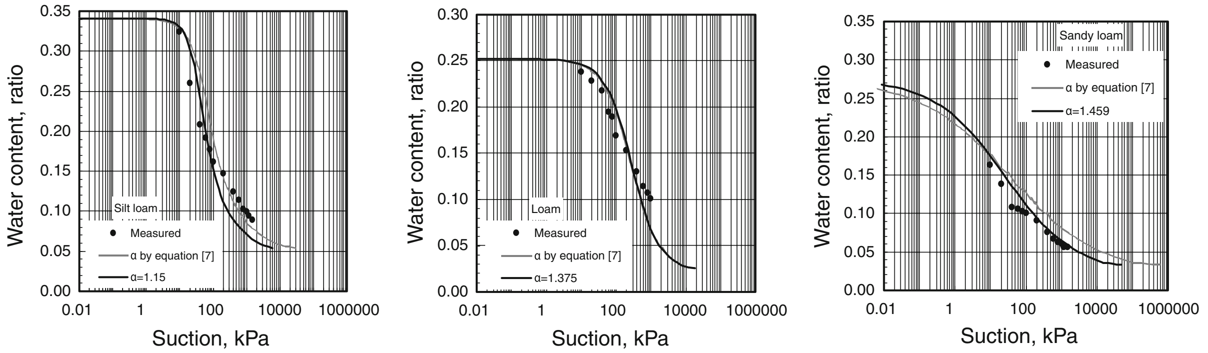


Fig. 3 Comparison of measured and predicted SWCC for selected soils, considering α as a constant and a continuous function

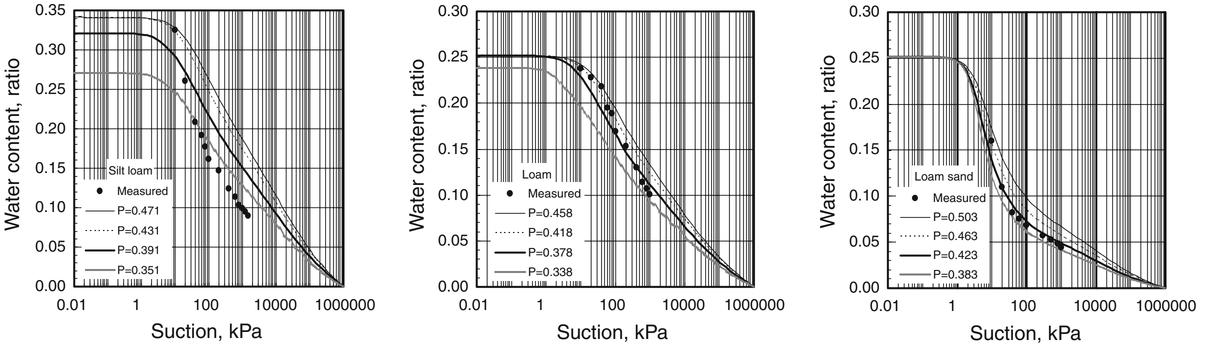
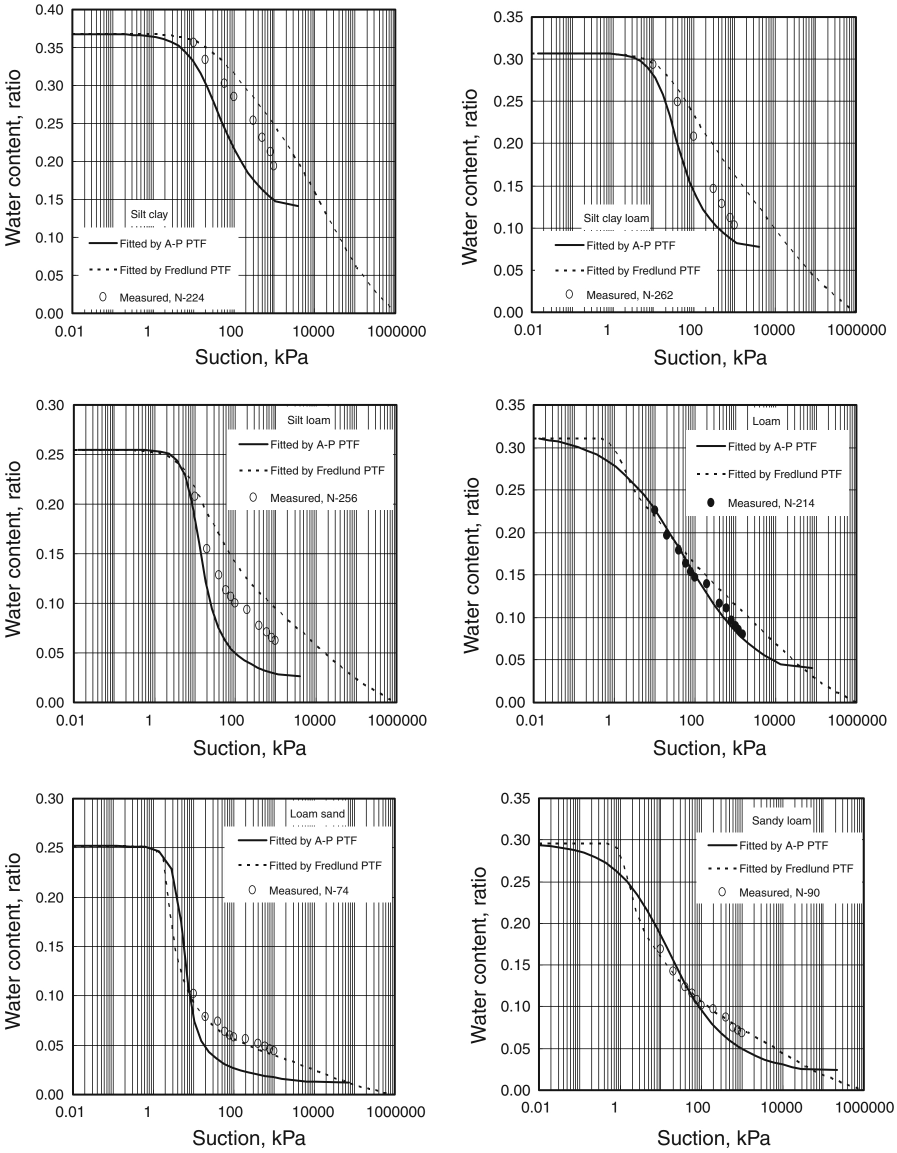


Fig. 4 Effect of packed porosity on prediction of SWCC for selected soils as example

Fig. 5 Comparison of predicted and measured SWCCs for six soil types using the Arya and Paris (1981) and Fredlund et al. (2002) PTFs



soils for both PTFs, although the predicted SWCCs look similar to the measured results.

Both PTFs were found to provide a reasonable estimate of the SWCC for loam soils and sandy–loam soils. For loam–sand soil, the Fredlund et al. (2002) PTF produced a reasonably accurate prediction of the SWCC, but the Arya and Paris (1981) PTF failed to predict SWCC. It has been previously noted that it is particularly difficult to estimate the SWCC from the particle-size distribution for some soil types. These general soil categories include: (1) soils that have a high amount of clay size particles, (2) soils that contain large amounts of coarse-size particles mixed with few fines, and (3) soils that exhibit bimodal particle-size distribution (Fredlund et al. 2002; Hwang and Powers 2003). The same trend was found to be true for loess soils.

A comparison is presented between measured and calculated water contents for 258 soil samples at the same soil suction using the Fredlund et al. (2002) and Arya and Paris (1981) PTFs. The results are shown in Fig. 6 and the statistical results for all samples and each soil type are given in Table 4. Based on Fig. 6 and statistical results in Table 4, it appear that the Fredlund et al. (2002) PTF provides a better estimation of the SWCC than the Arya and Paris (1981) PTF for the loess soils used in this study. For all samples, the Fredlund et al. (2002) PTF resulted in a lower RMSE of 0.039 cm^3 cm^{-3}, and a higher regression coefficient of 0.878. The RMSE for the Arya and Paris (1981) PTF was 0.046 cm^3 cm^{-3} and the R^2 was 0.768. The Arya and Paris (1981) PTF shows a consistent over-estimation of water contents in the low suction ranges, and an under-estimation of water contents in the high suction ranges. This resulted in a slope of 1.206, (i.e., larger than a perfect value of 1.0), and a low interception of -0.037 (i.e., less than a perfect value of 0.0) (Table 4). Amongst the six soil types, the Arya and Paris (1981) PTF provided a superior estimation of water contents for the sand–loam soils and the loam–sand soils, with a lower RMSE of 0.025 cm^3 cm^{-3} and 0.039 cm^3 cm^{-3}, respectively.

The Fredlund et al. (2002) PTF produced reasonably close predictions of the SWCC for sand–loam, loam–sand, and loam soils. The results showed a lower RMSE ranging from 0.006 to 0.036 cm^3 cm^{-3}, a high regression coefficient ranging from 0.922 to 0.970, and a lower interception from 0.001 to 0.022 cm^3 cm^{-3}, respectively. Over-estimations of the water contents were consistently observed for silt–loam, silt–clay–loam, and slit–clay soils with RMSE values ranging from 0.037 to 0.043 cm^3 cm^{-3}, the regression coefficient ranged from 0.805 to 0.868, and the interception ranged from 0.054 to 0.079 cm^3 cm^{-3}. These values are better than the estimation based on the Arya and Paris (1981) PTF.

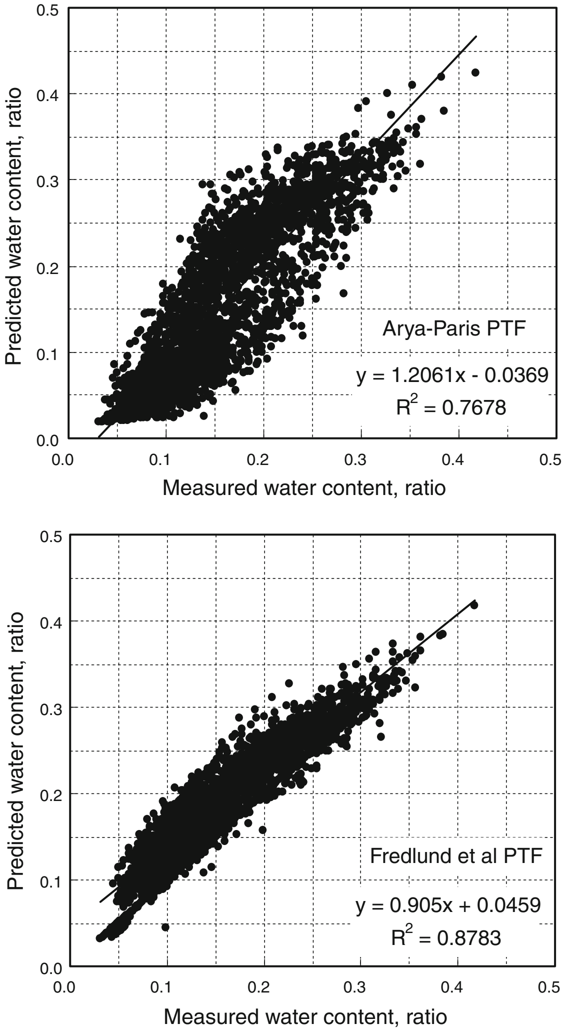


Fig. 6 Comparison between the measured and predicted gravimetric water content at the same soil suction by both PTFs

3.4 Comparison of Air-Entry Value

The air-entry value of the soil is the most relevant parameter associated with the SWCC. The air-entry value is the most important variable to determine for

Table 4 Comparison of the calculated and measured water contents for each soil type by the Ayra–Paris and Fredlund et al. (2002) PTFs

Soil type	No.	Linear regression			RMSE
		Interception	Slope	R^2	
Arya and Paris (1981) PTF					
Silt loam	2,270	−0.041	1.225	0.754	0.047
Loam	509	−0.037	1.267	0.791	0.049
Silt clay loam	109	−0.010	1.006	0.758	0.048
Sand loam	118	−0.045	1.299	0.848	0.039
Silt clay	63	−0.011	1.002	0.727	0.048
Loam sand	31	−0.029	1.334	0.758	0.025
All	3,101	−0.037	1.206	0.768	0.046
Fredlund et al. (2002) PTF					
Silt loam	2,270	0.054	0.868	0.880	0.040
Loam	509	0.029	0.970	0.871	0.036
Silt clay loam	109	0.063	0.835	0.904	0.043
Sand loam	118	0.022	0.922	0.875	0.025
Silt clay	63	0.079	0.805	0.923	0.046
Loam sand	31	0.001	0.969	0.956	0.006
All	3,101	0.046	0.905	0.878	0.039

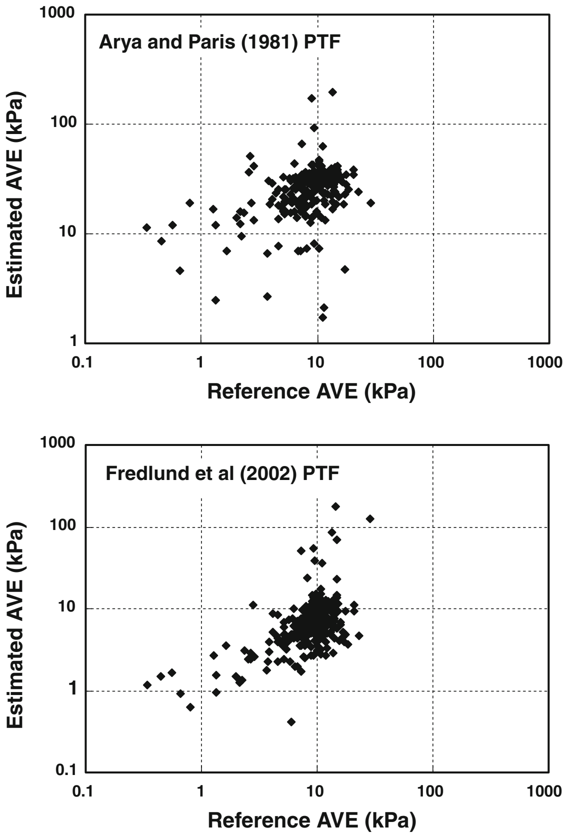


Fig. 7 Difference between measured and estimated air-entry values (*AVE*) for both PTFs

saturated–unsaturated seepage modeling in soil physics and geotechnical engineering. The two PTFs were evaluated on their ability to estimate the air-entry value for each soil. The reference air-entry value for each soil was determined from a best-fit regression on the experimental data. In each case, the Fredlund and Xing (1994) equation was best-fit to the SWCC data. The air-entry value for each PTF was calculated by the construction procedure published by Vanapalli et al. (1998, 2007). The comparisons of the estimated air-entry values from two PTFs and the experimental SWCC data for all soils are shown in Fig. 7. There is considerable scatter in the values estimated from both PTFs. Figure 7 shows that most of the air-entry values for the soils under consideration vary from 3.0 to 20.0 kPa. For this range of air-entry values, the Arya and Paris (1981) PTF show more values that are above the reference values. The Fredlund et al. (2002) PTF shows more values that are below the reference values. The MD and RMSE values for the air entry values produced by both PTFs are presented in Table 5. The estimated air-entry values were larger than the measured values for six soil textures when using the Arya and Paris (1981) PTF, and this resulted in a RMSE value of 25.41 kPa for all samples. The estimated air-entry values were larger than the measured values only for silt–clay–loam soils and silt–clay soils when using the Fredlund et al. (2002) PTF. For other soil types, the air-entry values estimated from the Fredlund et al. (2002) PTF were less than the measured values. The Fredlund et al. (2002) PTF appears to have greater accuracy in estimating the air-entry values for a soil. The overall RMSE value was 14.16 kPa for the Fredlund et al. (2002) PTF. The measured and estimated air-entry values showed that the air-entry value increases with the increasing clay content in the soil (Table 5).

3.5 Comparison of Maximum Slope

The rate at which a soil desaturates is another important soil parameter in assessing unsaturated soil hydraulic properties. Both PTFs were evaluated on their ability to estimate the rate at which a soil desaturates as suction increases. The representation

Table 5 Comparison of the estimated and measured air-entry values for all six textures

Soil types	Mean	SD	MD	RMSE
Measurement				
Silt–loam	9.64	3.42		
Loam	6.74	4.91		
Silt–clay–loam	12.70	5.45		
Sand–loam	5.59	3.34		
Silt–clay	11.29	2.20		
Loam–sand	4.87	3.07		
All	9.32	3.96		
Arya–Paris PTF				
Silt–loam	28.93	12.79	−19.29	22.96
Loam	24.68	34.19	−17.94	37.59
Silt–clay–loam	29.44	7.37	−16.74	19.55
Sand–loam	23.63	12.88	−18.04	21.93
Silt–clay	38.05	27.51	−26.76	37.37
Loam–sand	20.96	20.09	−16.09	22.53
All	28.43	17.24	−19.11	25.41
Fredlund et al. (2002) PTF				
Silt–loam	6.83	4.09	2.81	5.67
Loam	4.81	3.31	1.93	3.99
Silt–clay–loam	28.14	33.49	−15.44	31.80
Sand–loam	3.00	1.57	2.59	3.40
Silt–clay	61.54	62.77	−50.25	75.08
Loam–sand	1.84	0.24	3.03	3.81
All	8.67	14.94	0.65	14.16

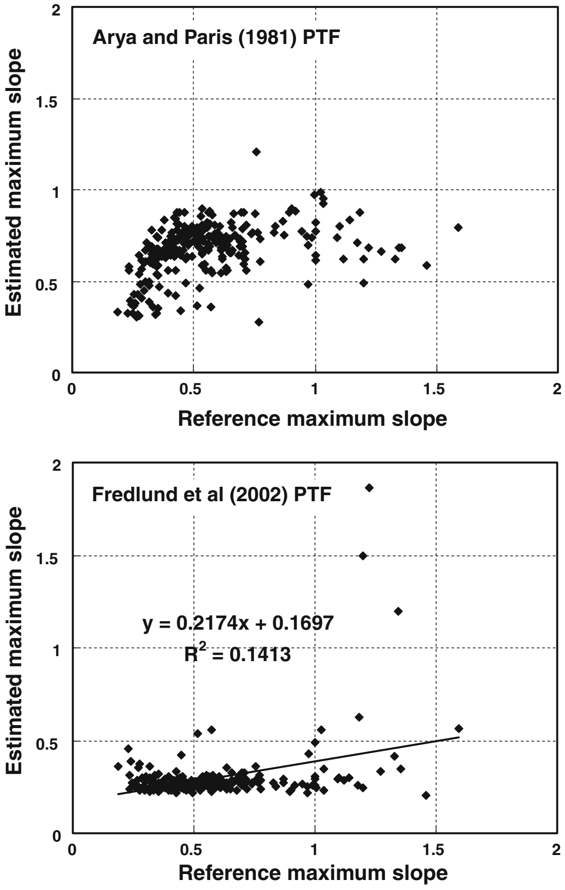


Fig. 8 Difference between measured and estimated maximum slopes for both PTFs

of the rate of desaturation was taken as the maximum slope on the SWCC (Fredlund et al. 2002) and was calculated as a change in gravimetric water content on the normalized SWCC divided by the change in the logarithm of soil suction in kPa. The maximum slope calculated when best-fitting the SWCC data with the Fredlund and Xing (1994) equation was taken as the reference value. The point of maximum slope corresponded to the inflection point on the best-fit curve. Each of the PTFs was evaluated by comparing the calculated and estimated maximum slope along the SWCC. The comparisons of the maximum slopes for both PTFs are shown in Fig. 8. Most of the maximum slope values were in the range from 0.2 to 0.7. The predicted maximum slopes from the Arya and Paris (1981) PTF are generally higher than the reference values. The predicted maximum slopes from the Fredlund et al. (2002) are generally lower than the reference values. The statistical results comparing the estimated and measured values for all soil types are presented in Table 6. From the Arya and Paris (1981) PTF, the estimated maximum slope shows greater accuracy than that obtained from the Fredlund et al. (2002) PTF. The RMSE values are 0.266 for the Arya and Paris (1981) PTF and 0.356 for the Fredlund et al. (2002) PTF, for all soil samples. In general, soils with a high sand content have a larger maximum slope.

4 Conclusions

The Fredlund et al. (2000) PSD and both PTFs, Arya and Paris (1981) and Fredlund et al. (2002), were evaluated using a 258 soil sample dataset measured on soils from the Loess Plateau in China. Each PTF estimation was compared with measured values.

Table 6 Comparison of the estimated and measured maximum slopes for all six textures

Soil types	Mean	SD	MD	RMSE
Measurement				
Silt–loam	0.569	0.218		
Loam	0.469	0.268		
Silt–clay–loam	0.414	0.201		
Sand–loam	0.709	0.421		
Silt–clay	0.406	0.229		
Loam–sand	1.321	0.285		
All	0.558	0.249		
Arya–Paris PTF				
Silt–loam	0.716	0.096	−0.147	0.259
Loam	0.506	0.146	−0.037	0.263
Silt–clay–loam	0.538	0.157	−0.124	0.236
Sand–loam	0.651	0.289	0.058	0.367
Silt–clay	0.526	0.198	−0.120	0.128
Loam–sand	0.821	0.153	0.500	0.599
All	0.677	0.141	−0.119	0.266
Fredlund et al. (2002) PTF				
Silt–loam	0.268	0.034	0.301	0.366
Loam	0.392	0.345	0.077	0.257
Silt–clay–loam	0.247	0.031	0.167	0.256
Sand–loam	0.388	0.059	0.321	0.477
Silt–clay	0.270	0.016	0.136	0.252
Loam–sand	0.771	0.367	0.550	0.659
All	0.291	0.144	0.267	0.356

Comparisons were made with respect to the estimation of gravimetric water content at the same soil suction, air-entry value, and maximum slope. The following observations can be drawn from this study:

(1) The Fredlund et al. (2000) PSD equation accurately represents the PSDs for all soils in the Loess Plateau, with a lower root mean square error (RMSE) of 0.869%.

(2) The performance of Arya and Paris (1981) PTF could not be significantly improved by using a continuous function to estimate the scaling parameter rather than using a constant. The packing porosity for the Fredlund et al. (2002) PTF does not always affect the SWCC estimations in the same way for different soil types.

(3) The Arya and Paris (1981) and Fredlund et al. (2002) PTFs were used to predict SWCC, and the statistical results showed that the Fredlund et al. (2002) PTF appeared to performed slightly better than the Arya and Paris (1981) PTF for most soils. Amongst the six soil types, the Fredlund et al. (2002) PTF produced the best SWCC predictions for sand–loam, loam–sand, and loam soils with a lower RMSE ranging from 0.006 to 0.036 $\mathrm{cm}^3\ \mathrm{cm}^{-3}$. The calculated water contents were consistently over-estimated by the SWCCs for silt–loam, silt–clay–loam, and slit–clay soils with RMSE values ranging from 0.037 to 0.043 $\mathrm{cm}^3\ \mathrm{cm}^{-3}$.

(4) Results showing the comparison between the measured and estimated air-entry values indicated a significant improvement when using the Fredlund et al. (2002) PTF. The estimated results from both PTFs showed that the air-entry value increases with increasing content of clay particles in soil.

(5) The maximum slope of the SWCC computed using both PTFs showed reasonable accuracy when compared with the maximum slope computed using the experimental data. The RMSE value between the experimental and measured results for all soil samples indicates that a better performance was obtained using the Arya and Paris (1981) PTF.

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