COMPARISON BETWEEN 3D LIMIT EQUILIBRIUM AND SHEAR STRENGTH REDUCTION METHODOLOGIES

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ABSTRACT: Limit equilibrium analysis of slopes has been commonplace in the geotechnical industry for many years. The 2D approach is conservative in that 3D geometric influences are not accounted for in the 2D analysis. The conservatism associated with a 2D analysis has been viewed as a “buffer” or added factor of safety but such conservatism can be problematic. Recent software tools allow for an improved analysis of 3D slopes through limit equilibrium analysis techniques. The industry as a whole must also examine the use of 3D analysis in light of how design expectations are managed.

The Shear Strength Reduction (SSR) technique calculates the factor of safety based on a FEM analysis of stresses. The advantage of the SSR method of analysis is that stresses and deformations of support elements such as piles, anchors, and geotextiles can be accounted for in greater detail in a slope stability analysis. Even given the apparent advantage, the SSR technique remains a new analysis in the geotechnical industry. The purpose of this paper is to compare 3D finite element stability analysis with 3D limit equilibrium analysis through the examination of benchmark examples. The classic differences between 2D finite element stress analysis and 2D limit equilibrium stress analysis will be examined. Continuity when going from 2D to 3D analysis is examined.
KEYWORDS: 3D Slope Stability Analysis, Shear Strength Reduction, Limit Equilibrium Method, Finite Element Method.

INTRODUCTION

2D limit equilibrium analysis (LEM) has been commonplace in geotechnical engineering practice over the past several decades. The method is easy to use and computationally quick to perform. The primary limitations associated with the LEM have been identified and have resulted in the 2D LEM coming under increased scrutiny. The limitations include; i) most slope failures are 3D in nature, ii) searching for the critical slip surface remains difficult, and iii) it is difficult to handle more advanced and real-world stress conditions such as K_o conditions in a numerical model.

The limitations associated with 2D LEM have led to increased interest in 3D models. As well, numerical models have been developed which better account for stress considerations. The 3D LEM method holds promise as it can also be modified to import a stress field from a finite element method (FEM) analysis. Advancements in computational capacity have also led to the ability to apply the shear strength reduction (SSR) techniques in 3D.

This paper presents results of an internal research program undertaken at SoilVision Systems on comparisons between 3D LEM methodologies as implemented in the SVSLOPE 3D software and the 3D SSR as implemented in the internal version of the SVSOLID 3D software. The comparisons are designed to explore the potential differences between the two methodologies. The comparisons are also designed to lead to increased confidence when applying either of these technologies in geotechnical engineering practice.

The 2D LEM is widely used in geotechnical engineering for slope stability analysis. However, slope failures are generally 3D in shape. The 2D approach is generally considered to be conservative in that 3D influences of geometry are not accounted for in a 2D analysis. Furthermore, the assumption that 2D analyses lead to conservative factors of safety is correct only when the critical section of the 3D model is selected for the 2D analyses. It is time consuming to ensure that the 2D section model is truly the critical 2D section for general slopes. The use of 3D slope stability analysis is important when modeling real world problems. 3D analyses make designs more economical and provide a guide for 2D designs. It is useful, for example, to know the percentage difference between 3D factors of safety and 2D factors of safety. The most common methods for 3D slope stability analysis are 3D LEM based on columns and 3D SSR based on FEM analysis.

3D LEM Slope Stability Analysis

3D LEM slope stability analysis is traditionally based on an extension of 2D LEM analysis. Many researchers have undertaken research on 3D LEM analysis (Hovland 1977, Zhang 1988, Hungr, Salgado and Byrne 1989, Lam and Fredlund 1993, Cheng, etc. 2005). The slicing method in 2D analyses has been extended into 3D analysis with columns by various authors due to the popularity of 2D LEM slicing methods. Some of the benefits of the 2D slicing method include its ability to accommodate complex geometries, variable soils, pore-water pressure conditions and various reinforcement systems.

Most 3D LEMs are based on the assumption that the failure direction for a slope is pre-defined in order to derive the factor of safety equations, (i.e. the failure sliding direction is not part of the slope stability analysis solution). Location of the critical failure surface and its direction is a challenging global optimization problem. Jiang (1997), Yamagami and Jiang (1997) provided an optimization-minimization procedure (OMP) known as Dynamic Programming (DP). Baker, (1980) used a random number generation technique to find the critical slip surface and corresponding sliding direction. Cheng and Yip (2003) derived 3D asymmetric slope stability analysis equations based on extensions of Simplified Bishop, Simplified Janbu and Morgenstern-Price methods. The direction of sliding was determined from 3D force/moment equilibrium equations and the formulation is equivalent to the Yamagami and Jiang (1997) OMP.

3D FEM-SSR Slope Stability Analysis

The finite element method (FEM) has been extensively used to analyze various geotechnical problems. To perform a slope stability analysis using the FEM, the SSR technique dictates that the soil shear-strength is gradually reduced until failure conditions occur. The factor of safety (FOS) for a SSR analysis is defined as the ratio of the shear strength of the soil to the shear stress developed along the critical failure surface and is mathematically written as follows.

$$c_f = c/SRF \tag{1}$$

$$\emptyset_f = \tan^{-1}(\tan\emptyset/SRF) \tag{2}$$

where c and $\emptyset$ are the cohesion and angle of internal friction for the Mohr-Coulomb shear strength parameters. c_f and $\emptyset_f$ are factored shear strength parameters. *SRF* is called the strength reduction factor. In order to reach to the state of limiting equilibrium, the *SRF* is gradually increased. This means that the soil shear strength gradually becomes weaker and weaker, until it is no longer possible for the FE model analysis to reach convergence. At this stage, the slope is said to fail and the factor of safety is equal to *SRF*. Non-convergence within a specified number of iterations and tolerance is an indicator of slope failure because of the absence of force equilibrium (i.e., stress and displacement distributions that satisfy the equations of equilibrium cannot be established based on the factored set of shear strength parameters).

The FEM-SSR analysis has been shown to be a powerful and a useful alternative to conventional LEM slope stability analytical techniques, (Griffiths *et al.,* 1999, 2007; Wei 2009). SoilVision Systems Ltd. has incorporated the 3D FEM-SSR into its commercial package called SVSOLID 3D (to be released shortly). Consequently, the 3D FEM-SSR will be readily available in geotechnical engineering practice.

EXAMPLES

The following benchmarks / examples are utilized to demonstrate the differences between various methods of analyzing the stability of a slope and the resulting factor of safety.

Example 1: A Conical Heap

The stability of a conical heap will be analyzed in this section. This example was first analyzed by Baker and Leshchinsky (1987) using the variational limit equilibrium approach and then analyzed by Jiang (1997) using LEM with dynamic programming. Figure 1 illustrates half of the extended log-spiral surface generated by the variational calculus solution.

In Baker and Leschinsky (1987), a non-dimensional parameter $\lambda = c/(\gamma H \tan\phi)$ was used to consider the effect of different parameters on the critical slip surface. A 3D model is shown in Figure 1. The non-dimensional parameter was λ set to 0.067. This lambda value, λ, corresponds to Figure 6 in Baker and Leshchinsky (1987) with the height of the cone at 5.0m. The unit weight of the material was 20 (kN/m^3), cohesion was 21 kPa, $\phi = 17.5$ degree. The comparison of factor of safety results calculated by various researchers is shown in Table 1.

Table 1. Comparison of 3D FOS for the conical heap in Example 1

3D LEM	**3D SSR**	**Jiang(1997) (LEM+DP)**	**Baker et al., (1087) (Variational Analysis)**
1.00	0.95	0.96	1.00

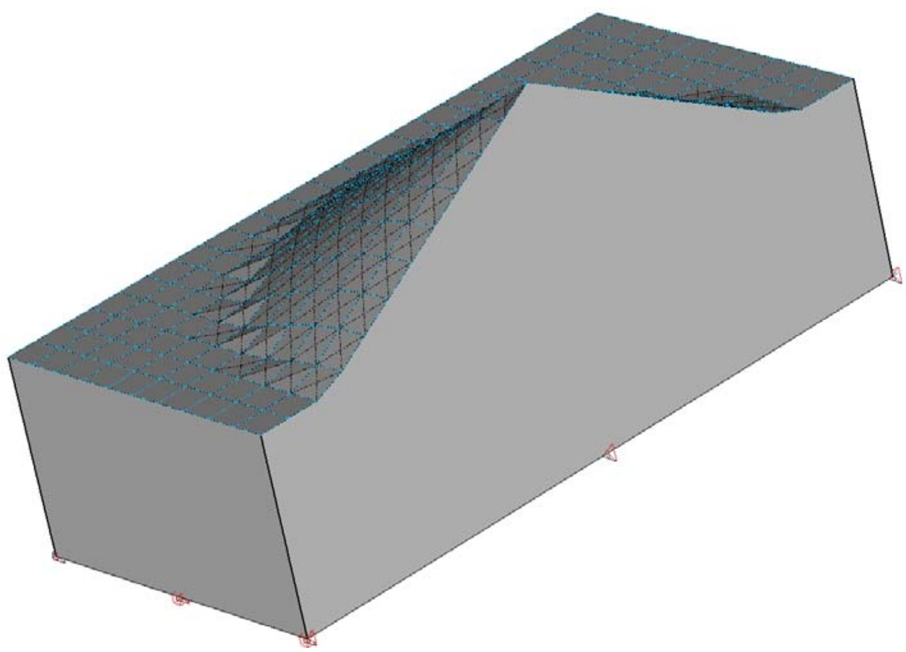

FIG 1. A Conical heap model

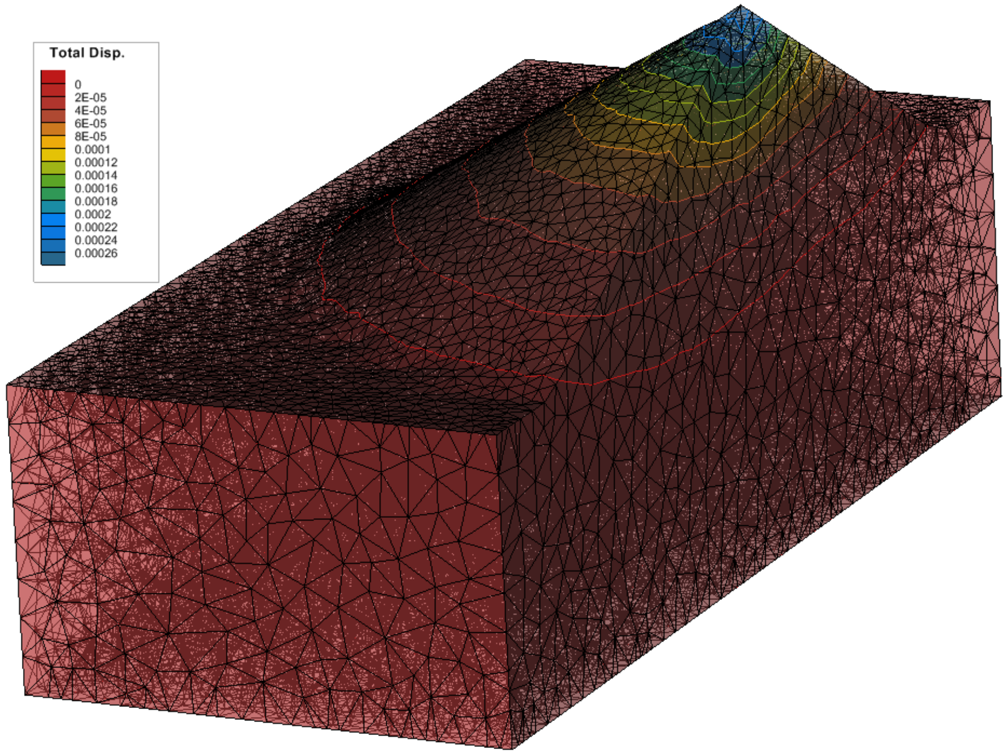

FIG 2. Contour of total displacement of 3D Shear Strength Reduction analysis

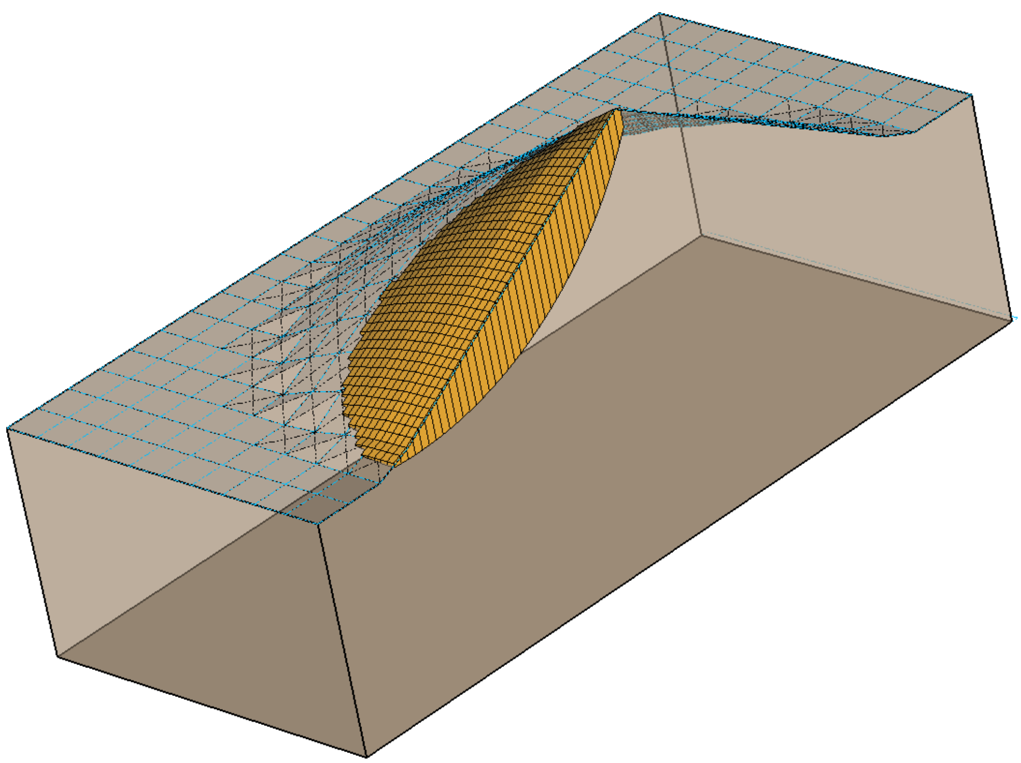

FIG 3. The critical slip surface of the 3D LEM analysis (Spencer Method)

Example 2: A 3D Slope with Complicated Geometry

A general 3D slope is considered in this example. The model was digitized from Jiang (1997). The plan view with contours of the ground surface elevation is shown in Figure 7. The material parameters used for this model are cohesion of 9.6 kPa, angle of internal friction of 15 degree, and a unit weight of 18.0 kN/m^3. The 3D slope shown in Figure 7 has a critical sliding direction that is unknown. In other words, the critical sliding direction needs to be calculated as part of 3D solution for LEM analysis.

One of the advantages of the 3D FEM-SSR analysis is that the sliding direction does not need to be specified in advance, while for the 3D column-based LEMs, the critical sliding direction is another variable which must be determined through a searching procedure. The ability to search for the critical slip surface sliding direction with optimization has been utilized in this study. The purpose of this example is to compare 3D FEM-SSR slope stability analysis and 3D LEM slope stability analysis for general slopes without pre-knowing the sliding direction information.

Table 3 shows the comparison from different analytical results. Jiang (1997) showed a factor of safety of 1.11 based on 3D Simplified Janbu method with the use of dynamic programming searching. The factor of safety from the FEM-SSR analysis was 1.15. The contours of the final total displacement at the final stage are shown in Figure 8. The critical slip mass based on 3D LEM analysis is shown in Figure 9. The factor of safety was 1.12 for the Simplified Bishop method. The critical slip direction also needed to be found. As shown in Figure 10, the critical slip surface direction is 58 degree clockwise from the negative *x*-direction. It can be seen that both the shape and the sliding direction are similar for the FEM-SSR analysis and the LEM analysis (Figure 8 and Figure 9).

Table 3. Comparison of 3D FOS for the slope in Example 2

3D LEM	3D SSR	Jiang(1997) (LEM+DP)
1.12 (Bishop)	1.15	1.11

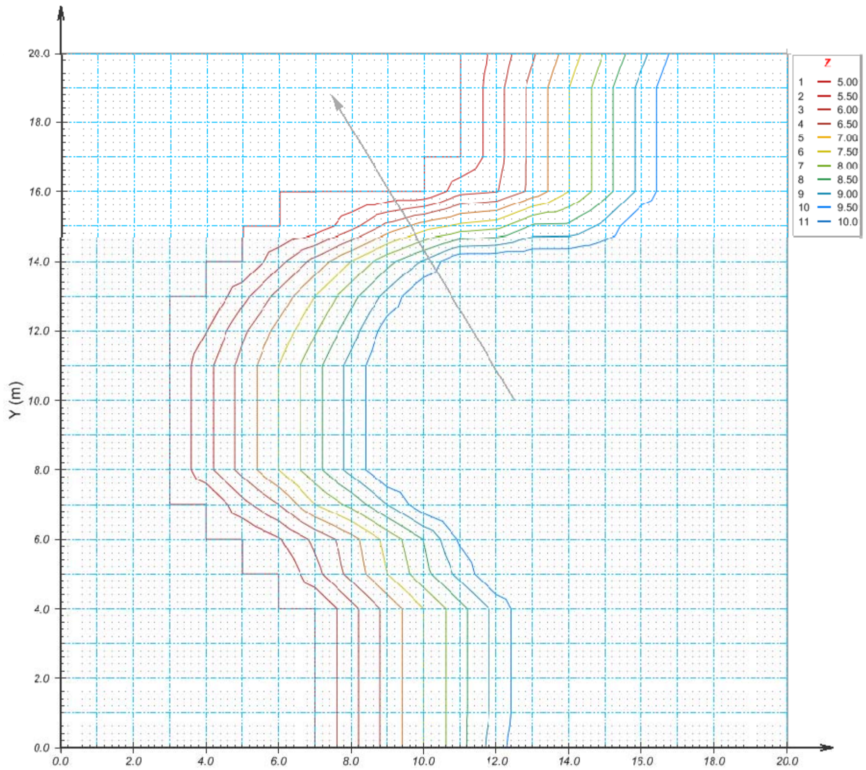

FIG 4. Plan view of the general slope with contour of elevation of the ground surface

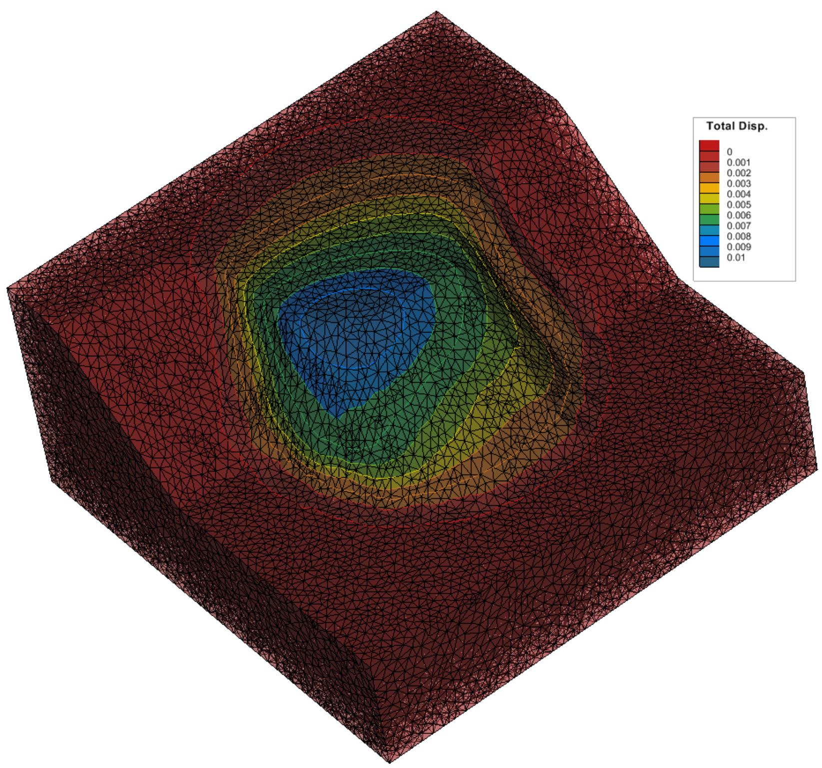

FIG 5. Contour of total displacement of 3D Shear Strength Reduction analysis

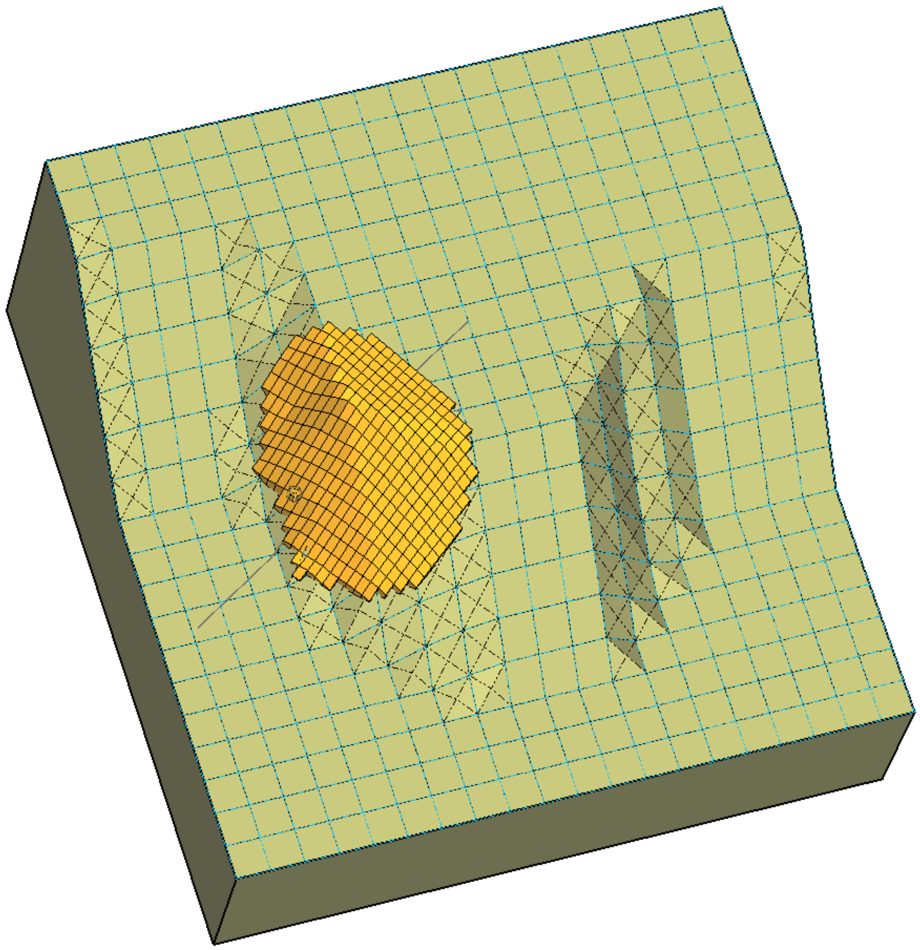

FIG 6. The critical slip surface of 3D LEM analysis result (Simplified Bishop method)

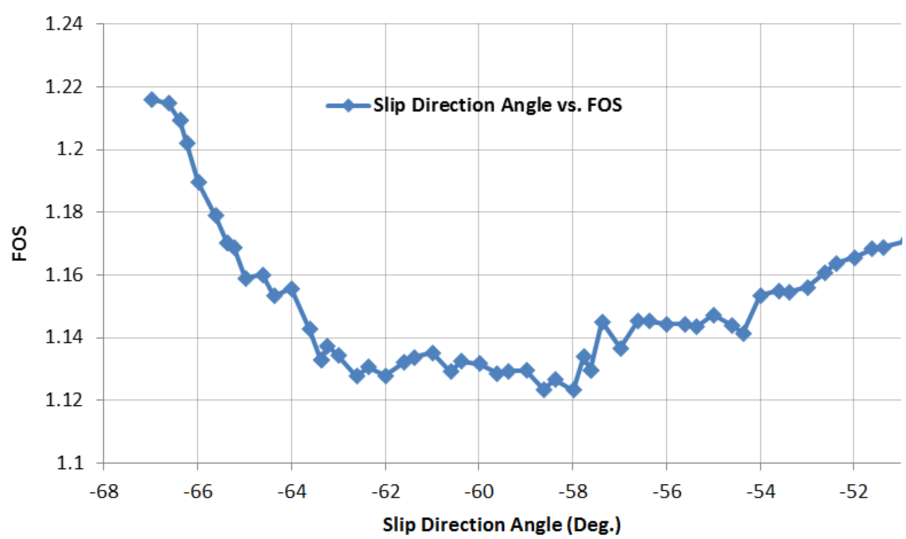

FIG 7. Plot of slip direction angle vs. FOS for Example 2

CONCLUSIONS

A slope failure occurs along the most critical sliding direction. This direction is often unknown in a general 3D slope stability analysis. Determination of the critical slip surface and the corresponding factor of safety involves the search for the critical sliding direction. One of the advantage of FEM-SSR technique is that it does not need to specified the sliding direction in advance, however it should also be noted that the method cannot provide the precise direction of sliding. An optimizaton technique that can be used to find the critical sliding direction as part of factor of safety search was utilized in this study.

The first example demonstrates the consistency amongst the LEM, SSR, LEM+Dynamic Programming methods, and the variational calculus analysis. The resulting factor of safety for each of the methods varies only between 0.95 and 1.00.

The second example investigates the case where the slip surface may be truncated by a bedrock layer. The analysis demonstrates consistent results between the LEM method (two software packages) and the SSR method. Both the location of the slip surface and the calculated factor of safety are consistent between the two methodologies.

Example 2 demonstrates the similarity of results between analyzing a complex 3D slope stability geometry where the direction of the slip may have an effect on the calulated factor of safety. A factor of safety of 1.12 was calculated when using the Bishop Simplified method and the value agrees well with other methods. It is also possible to determine the most likely slip direction of 58 degrees through an interative searching technique.

These results demonstrate the value of being able to use both the 3D LEM and the FEM-SSR methodologies for the analysis of slope geometries and loading conditions which are 3D in character.

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