Estimation of SWCCs from Grain-Size Distribution Curves for Loess Soils in China

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ABSTRACT
Indirect pedo-transfer functions, (PTFs) are increasingly being used for the estimation of the soil-water characteristic curve, SWCC. The accuracy of the PTF method depends on the PTF and the equation used to best-fit the particle-size distribution (PSD). The objectives of this study are to: 1.) evaluate the performance of the Fredlund et al. (2000) equation for best-fitting the PSD data, and, 2.) compare the predictions made by two of the commonly used PTFs; namely, Arya and Paris (1981) and Fredlund et al. (2002), for estimating the SWCC from the PSD. The authors used 258 measured PSD and SWCC datasets from the Loess Plateau, China. The comparison between the estimated and measured SWCCs showed that showed that both PTFs performed quite well with the Fredlund et al. PTF performed somewhat better than the Arya-Paris function.

RÉSUMÉ
Les fonctions de pedo-transfert (PTFs) indirectes sont de plus en plus utilisées pour l'estimation de la courbe caractéristique d'eau-sol, (SWCC). L'exactitude de la méthode de réalisation de la PTF dépend du PTF et de l'équation utilisée pour mieux lisser la distribution de grandeur de particule (PSD). Les objectifs de cette étude sont : 1.) évaluer la performance de l'équation Fredlund et al. (2000) pour mieux lisser les résultats PSD et, 2.) comparer les prédictions faites par deux PTFs communément utilisés pour estimer le SWCC a partir du PSD; c'est a dire, Arya et Paris (1981) et Fredlund et al. (2002). Les auteurs ont utilisé 258 PSD's mesurées et les datasets SWCC du Plateau de Lœss, Chine. La comparaison entre le SWCCs estimé et mesuré a montré que les deux PTFs fonctionnent tout à fait bien; bien que celui de Fredlund et al. donna des résultants un peu mieux que celle d'Arya-Paris.

1 INTRODUCTION

The description and mathematical simulation of water movement through unsaturated soils requires soil properties known as hydraulic conductivity, (or coefficient of permeability), and water storage. The field and laboratory determination of hydraulic conductivity and water storage for unsaturated soils are laborious and costly (van Genuchten and Leiji, 1992). This has led to the development and use of indirect methods known as Pedo-transfer functions, PTFs. A variety of PTFs have been developed based on soil particle-size distributions, PSDs and other basic geotechnical properties. The capillary theory has been used to relate void space between particles to their ability to retain water (Arya and Paris, 1981).

Pedo-transfer functions can be categorized into two groups based upon estimation technique. The first group of PTFs uses statistical estimates of soil properties to describe the SWCC. The soil properties are particle-size and volume-mass properties. The second group of PTFs utilize a physico-empirical approach that converts particle-size distributions into pore-size distributions. The pore-size distributions are then used to develop a SWCC (Arya and Paris 1981; Aubertin et al. 2003).

It has been found that the performance of Pedo-transfer functions is largely dependant on the dataset used for the calibration of the model (Schaap and Leij, 1998). Inaccurate predictions often occur when predictions are made for soils that are outside the range of soils that were used for calibrating the Pedo-transfer function (Cornelis et al. 2001; Hodnett and Tomasella, 2002). The predicted SWCC often falls off to zero volumetric water content before the experimental data are completely desaturated. Fredlund et al. (2002) developed a method to predict entire SWCC using particle-size distribution curves. A packing factor was incorporated to represent soil porosity (or the packing between particles).

There are two independent measurements required in the verification of Pedo-transfer functions; namely, the particle-size distribution, PSD, and the soil-water characteristic curve, SWCCs. Increased confidence is required for the relationship of the PTF to the SWCC. One way to improve accuracy when using the PTF methodology is to use a mathematical equations to best-fit the PSD data sets. Fredlund et al. (2000, 2002) modified the Fredlund and Xing (1994) SWCC equation to permit the fitting of a continuous function that extends to the extremes of PSDs.

The indirect method for predicting the SWCC depends on the estimated PTF and the equation for the PSD data. The objectives of this study are to: 1.) evaluate the performance of the Fredlund et al. (2000) equation for best-fitting PSD curves, and, 2.) compare the predictions made by two of the commonly used PTFs; namely, Arya and Paris (1981) and Fredlund et al. (2002), for estimating the SWCC from the PSD. A dataset consisting of 258 soil samples from the Loess Plateau, China consisted of PSD and SWCC measurements. The dataset included 187 silt-loam soils, 41 loam soils, 11 silt-clay-loam soils, 10 sand-loam soils, 6 silt-clay soils, and 3 loam-sand soils.

2 MATERIALS AND METHODS

2.1 Fredlund et al. (2000) Equation for PSD

The PSD model selected for this study is the Fredlund et al. (2000) unimodal and bimodal model. The bimodal PSD model showed advantages over the unimodal model in some cases. However, the total number of fitting parameters is doubled when using the bimodal model. The number of experimental PSD data points for each soil used in this study was only six. Therefore, the parameters for the bimodal PSD model cannot be justified and only the unimodal PSD model was used (Fredlund et al. 2000).

2.2 Pedo-transfer Functions

There are a variety of PTFs in the literature by which the SWCC can be estimated from a PSD. However, only the Arya and Paris (1981) and Fredlund et al. (2002) PTFs were selected to estimate SWCCs.

2.3 Site Description

All samples comprising the soil data base for this study were taken from the Loess Plateau in China. The Loess Plateau is located in the upper and middle reaches of the Yellow River. The loess cover has a thickness ranging from 30 to 180 m. The region where the samples were taken is a transitional zone between the southeastern humid monsoon climate and the northwestern continental dry climate. When moving from the southeast to the northwest, the soil texture changes from loam-clay to sandy-loam soils. Silt loam soils cover about 90% in of the Loess Plateau. The silt content ranges from 60-75%.

2.4 Soil Sampling Strategy

The 258 samples were taken from a depth of 30-60 cm below ground surface in the woodland, grassland, and farmland in 86 counties of the Loess Plateau. Samples were collected for the measurement of soil bulk densities and SWCCs. The samples were collected in duplicate using a 100 cm^3 coring tool from two test pits dug at each site. Particle size analyses were also performed.

SWCCs measured using the pressure plate method (Smith and Mullins, 1991) in which the water content was measured for matric suctions of 10, 20, 40, 60, 80, 100, 400, 600, 800, 1000, 1200, and 1500 kPa on the desorption curve. The equilibrium water content corresponding to each applied pressure was measured.

Particle size distribution curves were measured using the pipette method (Gee and Bauder, 1986). The measured soil bulk densities and particle-size distributions were input to the SoilVision software for calculating the PSD curve and estimating the SWCC (See Table 1).

Table 1. The soil types and their mean particle contents.

Soil type	No.	Particle content (%)		
		Clay	Silt	Sand
Silt -loam	187	20.4	62.9	16.7
Loam	41	18.3	47.9	33.8
Silt-clay - loam	11	36.4	54.6	9.0
Sand- loam	10	10.8	28.9	60.3
Silt-clay	6	27.9	52.8	9.3
Loam -sand	3	7.8	20.2	72.0

[a], STD means Standard deviation

2.5 Statistical Analysis

Several statistical parameters were used to quantify the difference between the measured and estimated values for both particle-size distribution and SWCCs. The root mean square error, RMSE, the intercept, slope and R^2 results of a linear regression are presented. The RMSE is defined as:

$$RMSE = [\frac{1}{N}\sum_{i=1}^{N}(E_i - M_i)^2]^{1/2} \quad [1]$$

where: E_i and M_i are predicted and measured values for the i^{th} observation, and N is the total number of measurements.

3 RESULTS AND DISCUSSIONS

3.1 Prediction of Particle Size Distribution

Figure 1 shows the measured and best-fit distributions for selected particle-size distributions. Table 2 contains the RMSEs and the wellness-of-fit values for each soil type. The calculated particle-size distributions from the Fredlund et al. (2000) equation are in good agreement with the measured data. Values of RMSE for the six soil types range from 0.599% to 1.400%. The R^2 values for the six soils ranged from 0.9961 to 0.9996, and the R^2 values on the slopes ranged from 0.993 to 0.999 (Table 2). All statistical results show that the Fredlund et al. (2000) equation can accurately represent the particle-size distributions for all soil types from the Loess Plateau. The differences in RMSE and regression coefficients for the various soil types might result from the different number of observations in each soil category. The largest number of observations for any soil type was 187× 6 points, and the smallest was 3× 6 points.

3.2 Effect of Model Parameter on Prediction of SWCC

The scaling parameter α is a key variable when using the Arya and Paris (1981) PTF to estimate the SWCC. Arya and Paris (1981) estimated pore lengths for various fractions of the particle-size distribution curve by summing the diameters of spherical particles in a particular size fraction. Pore lengths based on spherical particles were scaled to natural pore lengths using a scaling parameter, α, which was found to have an average value of 1.38. Later investigations by Arya et al. (1982) showed that the average α value varied from one soil textural classes to another.

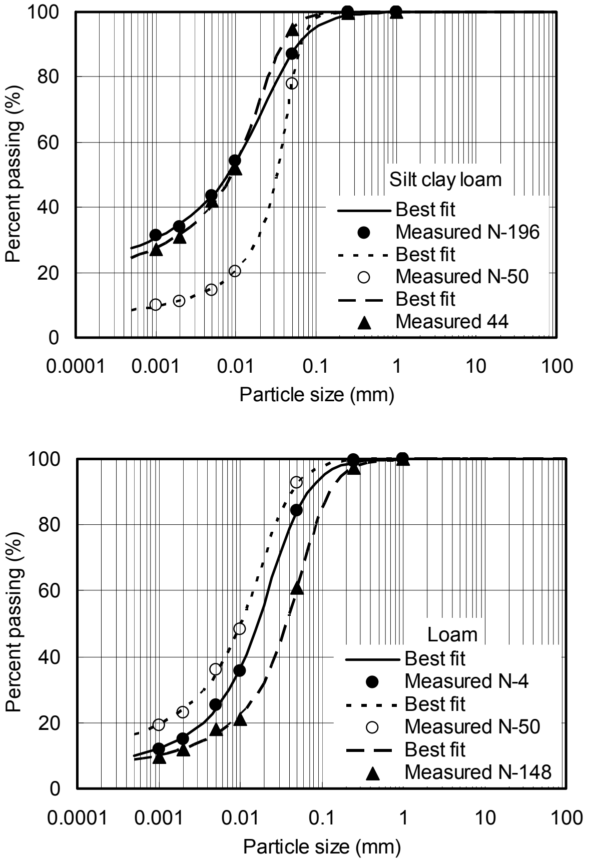

Figure 1. Prediction of particle-size distribution using the Fredlund et al. (2000) equation for selected soils

The α value ranged from 1.1 for fine-textured soils to 2.5 for coarse-textured soils. Arya et al. (1999) carried out further investigations and found that α was not constant but decreased with increasing particle diameters. An empirical formulation was developed for the estimation of suitable α values.

The effect of a constant value of α versus a continuous function, on the prediction of the SWCCs was studied by Arya et al. (1999). The final results showed that considering α as a continuous function could not significantly improve the predictive accuracy of SWCCs after 23 soils were tested (Arya et al. 1999). The effect of having α as a constant or a continuous function on the prediction of the SWCC prediction was also studied for 2 soil textures; namely, silt-loam soil and loam soil. The measured and predicted SWCC for 2 soils are presented in Figure 2. For loam soil, a constant α and a continuous function for α resulted in similar prediction of SWCC. For silt-loam soil, however, there was a larger error that occurred in the predicted SWCCs when α was considered as a continuous function. Consequently, in this study, the α variable was assumed as constant for each texture, and their values from Arya and Paris (1981).

Table 2: Comparison of the calculated and measured percent passing of particle sizes for all soil types when using the Fredlund et al. (2000) PSD equation.

Soil type	No.	Linear regression			RMSE (%)
		Int.^{b}	Slope	R^2	
Silt -loam	1122	0.078	0.998	0.9996	0.699
Loam	246	0.285	0.994	0.9986	1.320
Silt-clay -loam	66	0.129	0.998	0.9996	0.599
Sand -loam	60	0.146	0.998	0.9990	1.130
Silt -clay	36	0.265	0.999	0.9961	1.400
Loam -sand	18	0.189	0.993	0.9995	0.827
All	1548	0.123	0.997	0.9994	0.869

^{b}, Interception

When using the Fredlund et al. (2002) PFT to predict the SWCC, the grain-size distribution curve was divided into *n* fractions of uniformly sized particles. It is possible that the summation of the pore volumes for individual particle-size fractions may be greater than the overall porosity for the combined soil fractions. Therefore, a packing factor, *P,* was assumed for each fraction of soil particles. Ideally, the packing factor should be a function of the particle sizes, but in the Fredlund et al. (2002) PFT, it was assumed to be a constant for all particle sizes. The optimal value for the packing factor for each group of particle sizes was estimated by fitting the predicted SWCC with measured values.

Figure 3 shows the effect of different packing factors, *P*, on the predicted SWCCs for three selected soils. The results show that the packing factor does not always affect the SWCC estimation in the same way. The effect of different *P* values on the prediction of the SWCC varies from one soil type to another. For a silt-loam soil with an increasing *P* value, the predicted gravimetric water contents for all suction ranges decreases significantly. For loam soils, the predicted gravimetric water contents do not show as significant a change as for silt-loam soils. For the loam-sand soil, an increasing *P* value does not affect the predicted water content in the range from zero to 1 kPa, and the predicted water contents significantly decrease beyond a suction of 1 kPa. Therefore, the packed porosity has a significant influence on the predicted SWCC. It is suggested that further research should be undertaken on the role of the packing factor on the prediction of the SWCC.

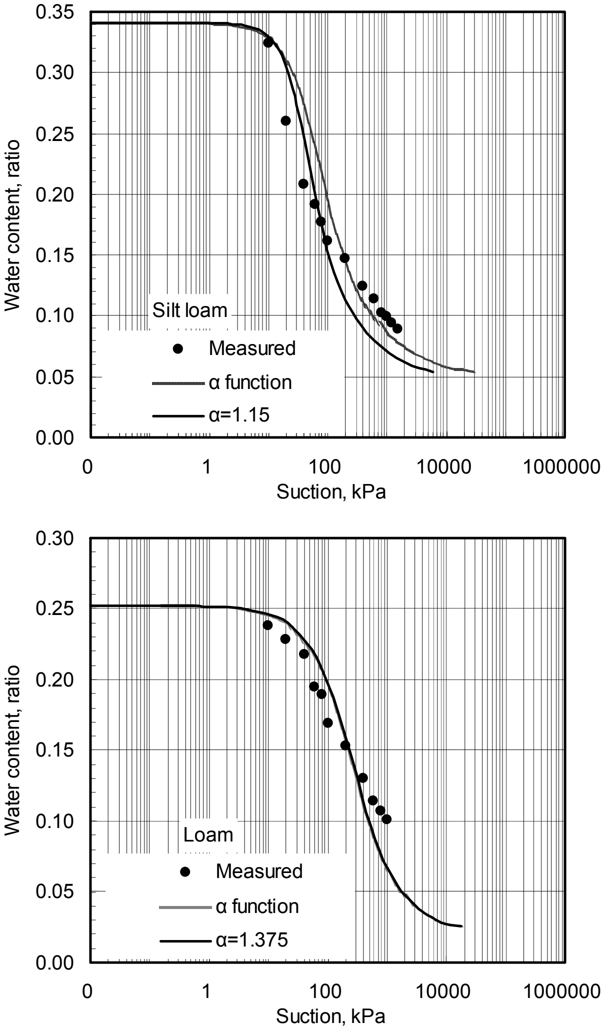

Figure 2. Comparison of measured and predicted SWCC for selected soils, considering α as a constant and a continuous function

3.3 The PTF Best-fit for SWCC

For 258 samples, the SWCCs were predicted using a constant α with the Arya and Paris (1981) PTF and the best-fit packing factor in Fredlund et al. (2002) PTF. There appears to be greater difficulty in estimating the SWCC for silt-clay soils, silt-clay-loam soils, and silt-loam soils for both PTFs, although the predicted SWCCs look similar to the measured results.

Both PTFs were found to provide a reasonable estimate of the SWCC for loam soils and sandy-loam soils. For loam-sand soil, the Fredlund et al. (2002) PTF produced a reasonably accurate prediction of the SWCC, but the Arya and Paris (1981) PTF failed to predict SWCC. It has been previously noted that it is particularly difficult to estimate the SWCC from the particle-size distribution for some soil types. These general soil categories include: (*i*) soils that have a high amount of clay size particles, (*ii*) soils that contain large amounts of coarse-size particles mixed with few fines, and (*iii*) soils that exhibit bimodal particle-size distribution (Fredlund et al. 2002; Hwang and Powers, 2003). The same trend was found to be true for loess soils.

A comparison is presented between measured and calculated water contents for 258 soil samples at the same soil suction using the Fredlund et al. (2002) and Arya-Paris (1981) PTFs. The results are shown in Figure 4 and the statistical results for all samples and each soil type are given in Table 3. Based on Figure 4 and statistical results in Table 3, it appear that the Fredlund et al. (2002) PTF provides a better estimation of the SWCC than the Arya-Paris (1981) PTF for the loess soils used in this study.

For all samples, the Fredlund et al. (2002) PTF resulted in a lower RMSE of 0.039 cm^3 cm^{-3}, and a higher regression coefficient of 0.878. The RMSE for the Arya-Paris (1981) PTF was 0.046 cm^3 cm^{-3} and the R^2 was 0.768. The Arya-Paris (1981) PTF shows a consistent over-estimation of water contents in the low suction ranges, and an under-estimation of water contents in the high suction range. This resulted in a slope of 1.206, (i.e., larger than a perfect value of 1.0), and a low interception of -0.037 (i.e., less than a perfect value of 0.0) (Table 3).

Amongst the six soil types, the Arya-Paris (1981) PTF provided a superior estimation of water contents for the sand-loam soils and the loam-sand soils, with a lower RMSE of 0.025 cm^3 cm^{-3} and 0.039 cm^3 cm^{-3}, respectively.

The Fredlund et al. (2002) PTF produced reasonably close predictions of the SWCC for sand-loam, loam-sand, and loam soils. The results showed a lower RMSE ranging from 0.006 to 0.036 cm^3 cm^{-3}, a high regression coefficient ranging from 0.922 to 0.970, and a lower interception from 0.001 to 0.022 cm^3 cm^{-3}, respectively.

Over-estimations of the water contents were consistently observed for silt-loam, silt-clay-loam, and slit-clay soils with RMSE values ranging from 0.037 to 0.043 cm^3 cm^{-3}, the regression coefficients ranged from 0.805 to 0.868, and the interception ranged from 0.054 to 0.079 cm^3 cm^{-3}. These values are better than the estimation based on the Arya and Paris (1981) PTF.

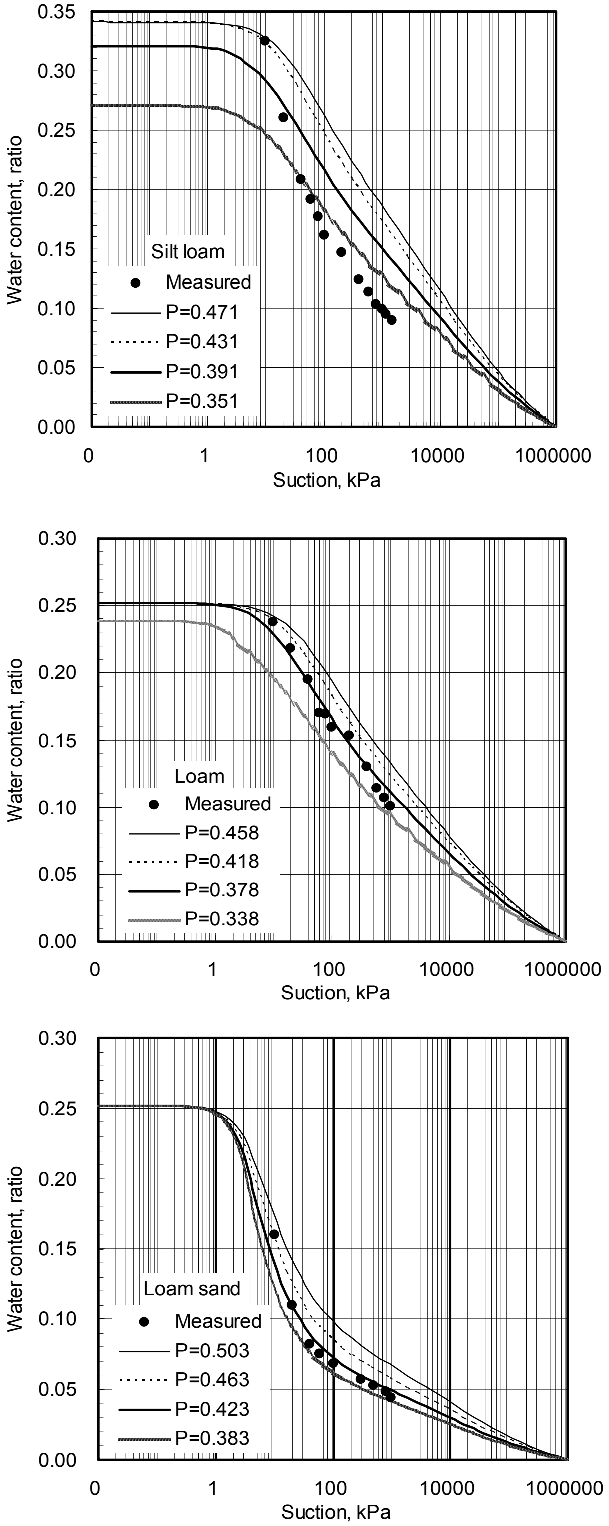

Figure 3. Effect of packed porosity on prediction of SWCC for selected soils as example

Table 3. Comparison of the calculated and measured water contents for each soil type by the Ayra-Paris and Fredlund et al. (2002) PTFs.

Soil type	No	Linear regression			RMSE
		Int.	Slope	R^2	
Arya-Paris (1981) PTF					
Silt-loam	2270	-0.041	1.225	0.754	0.047
Loam	509	-0.037	1.267	0.791	0.049
Silt-clay -loam	109	-0.010	1.006	0.758	0.048
Sand -loam	118	-0.045	1.299	0.848	0.039
Silt-clay	63	-0.011	1.002	0.727	0.048
Loam -sand	31	-0.029	1.334	0.758	0.025
All	3101	-0.037	1.206	0.768	0.046
Fredlend et al. (2002) PTF					
Silt-loam	2270	0.054	0.868	0.880	0.040
Loam	509	0.029	0.970	0.871	0.036
Silt- clay -loam	109	0.063	0.835	0.904	0.043
Sand -loam	118	0.022	0.922	0.875	0.025
Silt- clay	63	0.079	0.805	0.923	0.046
Loam -sand	31	0.001	0.969	0.956	0.006
All	3101	0.046	0.905	0.878	0.039

3.4 Comparison of Air-entry Value

The air-entry value of the soil is the most relevant parameter associated with the SWCC. The air-entry value is the most important variable to determine for saturated-unsaturated seepage modeling in soil physics and geotechnical engineering. The two PTFs were evaluated on their ability to estimate the air-entry value for each soil. The reference air-entry value for each soil was determined from a best-fit regression on the experimental data. In each case, the Fredlund and Xing (1994) equation was best-fit to the SWCC data. The air-entry value for each PTF was calculated by the construction procedure published by Vanapalli et al. (1998). The comparisons of the estimated air-entry values from two PTFs and the experimental SWCC data for all soils are shown in Figure 5. There is considerable scatter in the values estimated from both PTFs.

Figure 5 shows that most of the air-entry values for the soils under consideration vary from 3.0 to 20.0 kPa. For this range of air-entry values, the Arya and Paris (1981) PTF show more values that are above the reference values. The Fredlund et al. (2002) PTF shows more values that are below the reference values. The calculated RMSE values for the air entry values produced by both PTFs indicated that the estimated air-entry values were bigger than the measured values for 6 soil textures when using the Arya and Paris (1981) PTF, and that the estimated air-entry values were larger than the measured values only for silt-clay-loam soils and silt-clay soils when using the Fredlund et al. (2002) PTF. For other soil types, the air-entry values estimated from the Fredlund et al. (2002) PTF were less than the measured values. The Fredlund et al. (2002) PTF appears to have greater accuracy in estimating the air-entry values for a soil. The measured and estimated air-entry values showed that the air-entry value increases with the increasing clay content in the soil.

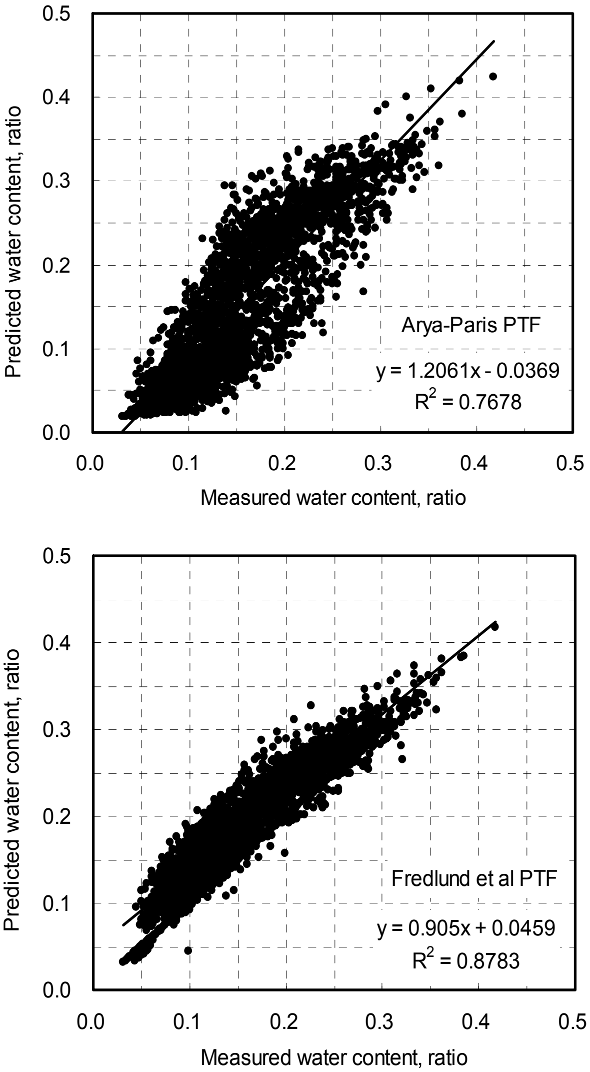

Figure 4. Comparison between the measured and predicted gravimetric water content at the same soil suction by both PTFs

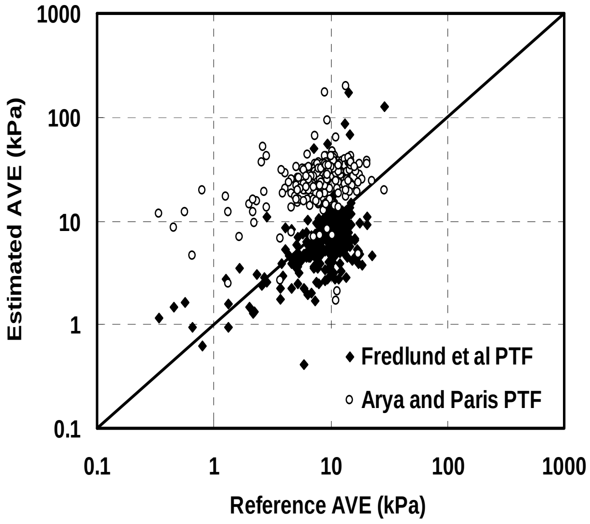

Figure 5. Difference between measured and estimated air-entry values (AVE) for both PTFs

3.5 Comparison of Maximum Slope

The rate at which a soil desaturates is another important soil parameter in assessing unsaturated soil hydraulic properties. Both PTFs were evaluated on their ability to estimate the rate at which a soil desaturates as suction increases. The representation of the rate of desaturation was taken as the maximum slope on the SWCC (Fredlund et al. 2002) and was calculated as a change in gravimetric water content on the normalized SWCC divided by the change in the logarithm of soil suction in kPa. The maximum slope calculated when best-fitting the SWCC data with the Fredlund and Xing (1994) equation was taken as the reference value. The point of maximum slope corresponded to the inflection point on the best-fit curve. Each of the PTFs was evaluated by comparing the calculated and estimated maximum slope along the SWCC. The comparisons of the maximum slopes for both PTFs are shown in Figure 6. Most of the maximum slope values were in the range from 0.2 to 0.7. The predicted maximum slopes from the Arya and Paris (1981) PTF are generally higher than the reference values. The predicted maximum slopes from the Fredlund et al. (2002) are generally lower than the reference values. The statistical results comparing the estimated and measured values for all soil types indicated that, from the Arya and Paris (1981) PTF, the estimated maximum slope shows greater accuracy than that obtained from the Fredlund et al. (2002) PTF. The RMSE values are 0.266 for the Arya and Paris (1981) PTF and 0.356 for the Fredlund et al. (2002) PTF, for all soil samples. In general, soils with a high sand content have a larger maximum slope.

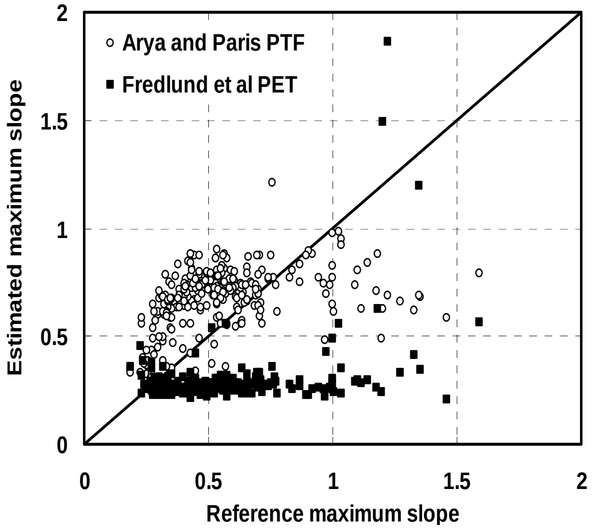

Figure 6. Difference between measured and estimated maximum slopes for both PTFs

4 CONCLUSIONS

The Fredlund et al. (2000) PSD and both PTFs, Arya and Paris (1981) and Fredlund et al. (2002), were evaluated using a 258 soil sample dataset measured on soils from the Loess Plateau in China. Each PTF estimation was compared with measured values. Comparisons were made with respect to the estimation of gravimetric water content at the same soil suction, air-entry value, and maximum slope. The following observations can be drawn from this study:

(1) The Fredlund et al. (2000) PSD equation accurately represents the PSDs for all soils in the Loess Plateau, with a lower root mean square error (RMSE) of 0.869%.
(2) The Arya-Paris (1981) and Fredlund et al. (2002) PTFs were used to predict SWCC, and the statistical results showed that the Fredlund et al. (2002) PTF appeared to performed slightly better than the Arya-Paris (1981) PTF for most soils.
(3) Results showing the comparison between the measured and estimated air-entry values indicated a significant improvement when using the Fredlund et al. (2002) PTF. The estimated results from both PTFs showed that the air-entry value increases with increasing content of clay particles in soil.
(4) The maximum slope of the SWCC computed using both PTFs showed reasonable accuracy when compared with the maximum slope computed using the experimental data. The RMSE value between the experimental and measured results for all soil samples indicates that a better performance was obtained using the Arya and Paris (1981) PTF.

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