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Estimation of volume change functions for unsaturated soils

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ABSTRACT: The estimation of unsaturated soil property functions has followed the pattern of using the saturated soil properties along with the soil-water characteristic curve to predict the behavior of the unsaturated soil. This has been the procedure for the permeability function and the shear strength functions for unsaturated soils. It has been more difficult to develop procedures for the volume change functions for an unsaturated soil; however, it is suggested that a similar procedure can be adopted. This paper suggests a series of postulates that can be used in estimating the volume change functions (during monotonic loading) for an unsaturated soil.

1 INTRODUCTION

Procedures to estimate the unsaturated soil property functions with respect to permeability and shear strength have been developed based upon the saturated soil properties and the soil-water characteristic curve. These procedures have proven to be of great value in the implementation of unsaturated soil mechanics into standard geotechnical engineering practice (Fredlund, 1999). Similar procedures are now required for the volume-mass constitutive relationships for an unsaturated soils.

2 ESTIMATION OF THE VOLUME-MASS FUNCTIONS

Over the past several decades, the volume change behavior of an unsaturated soil has been linked to two independent stress state variables; namely, $(\sigma - u_a)$ and $(u_a - u_w)$. This constitutive formulation forms the basis for modeling the volume change of an unsaturated soil. Modeling of such soil processes as stress/deformation, shrink/heave, and consolidation require an adequate description of the constitutive volume change behavior of a soil.

The overall volume change of an unsaturated soil can be defined as a change in void ratio in response to a change in the stress state (Fredlund and Morgenstern, 1976).

$$de = \frac{\partial e}{\partial(\sigma - u_a)} d(\sigma - u_a) + \frac{\partial e}{\partial(u_a - u_w)} d(u_a - u_w) \tag{1}$$

where: e = void ratio; and σ = total normal confining stress (e.g., isotropic confining pressure).

Equation (1) can be viewed as having two parts; namely, a part that is the designation of the stress state (i.e., $(\sigma - u_a)$ and $(u_a - u_w)$) and a part that is a designation of the soil properties (i.e., $(\partial e / \partial(\sigma - u_a))$ and $(\partial e / \partial(u_a - u_w))$). The soil properties can be viewed as the slope of the void ratio constitutive surfaces as shown in Figure 1. The soil properties are moduli that vary as a function of the stress state. The soil moduli associated with the net normal stress, $(\sigma - u_a)$, can be written in a general functional form.

$$\partial e / \partial(\sigma - u_a) = func[(\sigma - u_a),(u_a - u_w)] \tag{2}$$

The term *func* means that the soil property is a function of the stress state. At a particular stress state, the compressibility modulus for the void ratio constitutive surface with respect to $(\sigma - u_a)$, can be designated as a constant.

$$\partial e / \partial(\sigma - u_a) = m_1^s \tag{3}$$

Similarly, the soil moduli associated with soil suction, $(u_a - u_w)$, can be written in a general functional form.

$$\partial e / \partial(u_a - u_w) = func[(\sigma - u_a),(u_a - u_w)] \tag{4}$$

At a particular stress state, the compressibility modulus for the void ratio constitutive surface with respect to $(u_a - u_w)$, can be designated as a constant.

$$\partial e / \partial (u_a - u_w) = m_2^s \qquad (5)$$

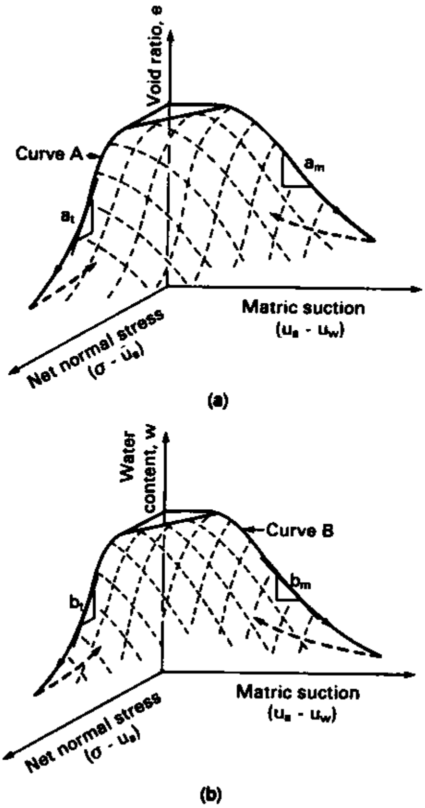

Figure1. Three-dimensional void ratio and water content constitutive surfaces for an unsaturated soil. (a) void ratio constitutive surface; (b) water content constitutive surface (Fredlund and Rahardjo, 1993).

Each of the soil moduli is function of both stress state variables. In order to define the magnitude of the soil moduli corresponding to any stress state, there needs to be a constitutive equation describing the entire void ratio constitutive surface. The equation then needs to be differentiated with respect to each of the stress state variables in order to obtain the compressibility moduli. At present, no equations have been published to represent the entire void ratio constitutive surface in terms of the stress state variables.

Two constitutive relationships are required to define the volume-mass variables in terms of the stress state variables. The need for two independent constitutive relations for an unsaturated soil can be demonstrated through the differentiation of the basic volume-mass relationship (i.e., $Se = wD_r$).

$$\int_{e_o}^{e_f} S de + \int_{S_o}^{S_f} e dS = D_r \int_{w_o}^{w_f} dw \qquad (6)$$

where: w = water content, with the subscript, o, and f, representing the initial and final states, respectively; S = degree of saturation; and D_r = relative density of the soil solids.

The water content constitutive surface can be used as a second relationship for defining the volume-mass behavior of an unsaturated soil (Fig. 1). The water content constitutive relationship can be written the following general form.

$$dw = \frac{\partial w}{\partial(\sigma - u_a)} d(\sigma - u_a) + \frac{\partial w}{\partial(u_a - u_w)} d(u_a - u_w) \qquad (7)$$

Once again, Equation (7) has a part that designates the stress state and a part that designates an unsaturated soil property that is a function of the stress state. The soil moduli associated with the net normal stress variable, $(\sigma - u_a)$, can be written as a general function.

$$\partial w / \partial(\sigma - u_a) = func[(\sigma - u_a),(u_a - u_w)] \qquad (8)$$

At a particular stress state, the compressibility modulus for the water content constitutive surface, with respect to $(\sigma - u_a)$, can be designated as a constant.

$$\partial w / \partial(u_a - u_w) = m_1^w \qquad (9)$$

Similarly, the soil moduli associated with the soil suction, $(u_a - u_w)$, can be written as a general function of the stress state.

$$\partial w / \partial(u_a - u_w) = func[(\sigma - u_a),(u_a - u_w)] \qquad (10)$$

At a particular stress state, the compressibility modulus for the water content constitutive surface, with respect to $(u_a - u_w)$, can be designated as a constant.

$$\partial w / \partial(u_a - u_w) = m_2^w \qquad (11)$$

At present, there is no published equation to represent the entire water content constitutive surface. Once an appropriate equation is formulated, the derivatives will provide the soil moduli values corresponding to any stress state.

3 FORMULATION FOR THE ESTIMATION OF THE VOLUME-MASS CONSTITUTIVE SURFACES (LOADING)

The formulation of the constitutive surfaces is based on common laboratory compression, shrinkage and soil-water characteristic curve tests. Mathematical representation needs to be proposed for the compression, shrinkage, and soil-water characteristic curve soil property functions. Numerous equations have already been proposed for the soil-water characteristic curve.

A series of assumptions are needed to form a guide for combining the independent mathematical representations and the subsequent formulation of void ratio and water content constitutive surfaces. (Only the assumption or postulates relevant to the volume change constitutive relationship are presented in this paper). The desired end result is a mathematical representation of two independent constitutive surfaces. The mathematical representations can then be used to compute the compressibility (and subsequently the elastic moduli) of the soil corresponding to any stress state in the soil. These formulations provide the necessary information for the generation of elastic soil property functions that can be used for numerical modeling (i.e., finite element method) of soil behavior.

3.1 Volume Change Constitutive Surface (Loading)

The overall volume change can be defined in terms of void ratio, e, or specific volume, υ, (i.e., 1+ e). The void ratio is used herein to define the first constitutive surface.

A series of "postulates" are proposed for the prediction of the volume-mass constitutive relations (Fredlund, 2000). The postulates establish a series of priorities that must be adhered to when attempting to estimate the volume-mass relationships. Certain information has become well established in the research literature and this information forms a series of hierarchical priorities when predicting the constitutive surfaces.

The soil structure constitutive surface can be defined as the relationship between two independent stress state variables and a deformation state variable. The independent stress state variables are: $(\sigma - u_a)$ = net normal (isotropic) confining pressure; and ψ = soil suction.

The deformation state can be defined in terms of void ratio, e. The proposed "postulates" for the soil structure (i.e., void ratio) constitutive surface are given below for the case of an increase in both of the stress state variables (i.e., a monotonic decrease in volume). In addition, it is assumed that the testing of the soil starts with the specimen being in a saturated state. There are a number of loading stress paths as well as wetting and drying paths that could be analyzed; however, it is important to start by developing constitutive surfaces for the conditions on which the most information is available.

Postulate 1: T*he **primary reference condition** for the volume change (overall) constitutive relationship is determined by applying a net (isotropic) total stress loading of the soil with the pore-water and pore-air pressures maintained at zero, while measuring the change in void ratio.*

This relationship is commonly referred to as the drained, effective stress loading path for a saturated specimen (Fig. 2). The term "isotropic" is placed in brackets to suggest that isotropic loading is the preferred form of loading. However, it is also possible for K_o or other forms of net total stress loading to be considered. Isotropic stress loading is preferable because: a) the stress path is the same as that used for critical state (or elasto-plastic) models, and, b) the matric suction stress state variable is also isotropic in character.

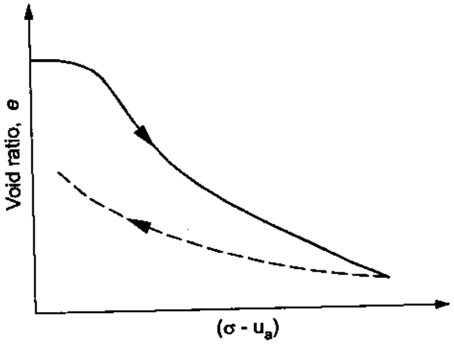

Figure 2. Typical loading and unloading curves of void ratio versus the applied load.

*Postulate 2: The **secondary reference condition** for the volume change (overall) constitutive relationship is determined by applying various soil suctions to the soil with the net isotropic stress equal to zero, while measuring the change in void ratio (Fig. 3).*

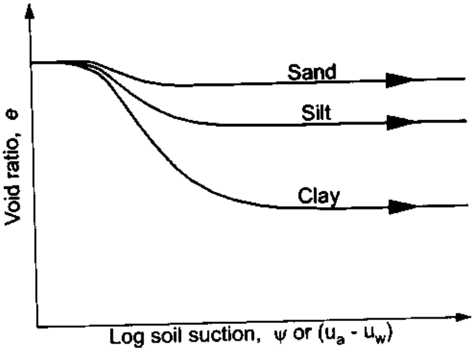

Figure 3. Typical void ratio versus soil suction plot for three soils (suction increasing).

There is a practical difficulty associated with directly measuring the volume change versus soil suction relationship. The difficulty is related to measuring volume change in three directions while changing soil suction. As a result of the above difficulty, "Postulate 2a" suggests an alternate means to indirectly provide the necessary secondary reference condition. An alternate procedure makes use of a combination of data from a shrinkage test and a soil-water characteristic curve test.

Postulate 2a: The void ratio versus soil suction relationship can also be computed using the soil-water characteristic curve for the soil along with the shrinkage curve, both sets of data are measured under condition of zero net isotropic stress.

Figure 4 shows three typical soil-water characteristic curves under drying conditions (or conditions of an increase in suction. Figure 4 shows a typical shrinkage curve associated with the drying of a clay soil.

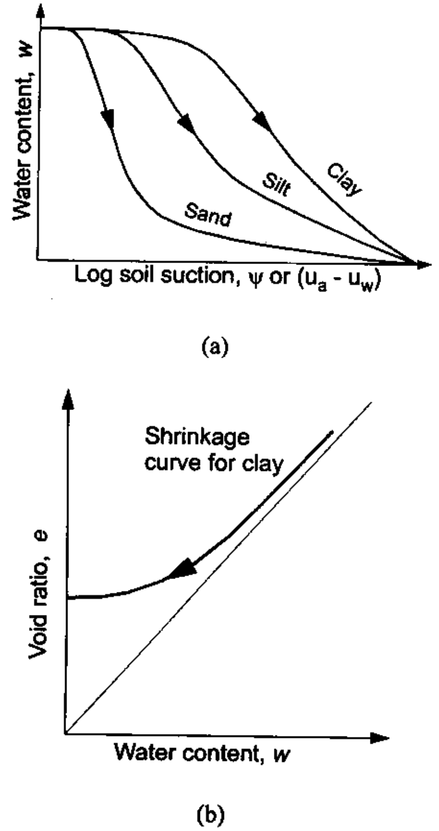

Figure 4. a) Typical soil-water characteristic curves for three soil types. b) Typical shrinkage curve for a clay soil.

It is possible to combine the results of a pressure plate test (i.e., soil-water characteristic curve data) and a shrinkage test to obtain a void ratio versus suction plot. The shrinkage test defines a curve that gives the ratio of change in volume of water to overall volume, for a change in soil suction. Mathematically, the slope of the shrinkage curve can be written as follows.

$$\frac{de}{dw} = \frac{de/d\psi}{dw/d\psi} = \frac{a_m}{b_m} \tag{12}$$

Combining the two sets of information makes it possible to compute the void ratio versus soil suction relationship. This forms the second reference (or limiting) condition for the soil structure constitutive surface.

Postulate 3: There is a unique volume change constitutive surface defined for conditions of monotonic deformation.

The surface for a decrease in volume under an increase in stresses is considered herein and shown in Figure 5.

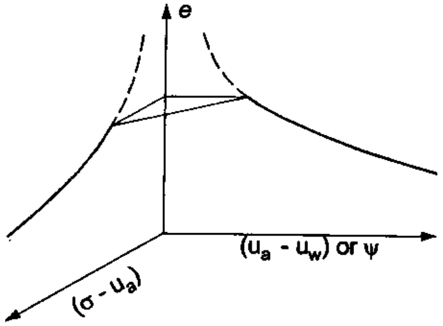

Figure 5. Three-dimensional plot showing the primary and secondary reference conditions for the void ratio constitutive surface.

The limiting (or reference) conditions associated with the void ratio constitutive surface have now been defined. The next step is to define the character of the constitutive surface between the limiting reference conditions. The remaining postulates pertain to establishing intermediate stress state conditions on the constitutive surface.

Postulate 4: The slope along any constant net total stress plane on the volume change constitutive surface is a function of the void ratio, as defined on the zero soil suction plane.

This postulate comes about as a result of Postulate 1 where it is stated that the void ratio versus net total stress is the primary and most fundamental relationship between void ratio and the stress state. As a consequence of Postulate 4, the slope of any line emanating from the soil suction versus void ratio curve, in a constant suction plane, must be equal to the compressibility defined on the primary reference curve at a corresponding void ratio. Appropriate slopes for the constitutive surface can be determined by constructing a triangle in the horizontal plane, between the reference conditions.

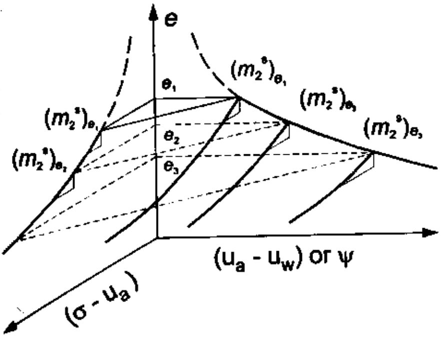

Figure 6. Illustration of the definition of the void ratio constitutive surface based on the slopes of the primary reference curve.

Postulate 5: There is a one-to-one relationship between the effects of changes in net total stress and a change in soil suction, when the soil suction is less than the air entry value of the soil (Fig. 7).

This mean that a 45 degree relationship will be defined between the two stress state variables when the void ratio constitutive surface is viewed along the void ratio axis.

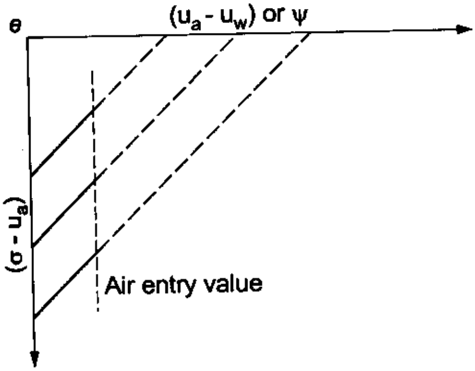

Figure 7. Variation of constant void ratio contours when the surface is viewed along the void ratio axis.

The straight line contour across the constitutive surface should be theoretically correct as long as the soil is saturated. This is in accordance with the effective stress concept for a saturated soil. It should be noted that the dashed lines drawn in Figure 7 may not intersect the secondary reference condition along the plane of net total stress equal to zero. It is necessary to comment further on the air entry value of the soil before suggesting a further refinement on void ratio contours.

Postulate 5a: As a first approximation, the air entry value of the soil can be assumed to be a constant, but for greater refinement, the air entry value may need to be defined as a function of void ratio or the net isotropic stress (Fig. 8).

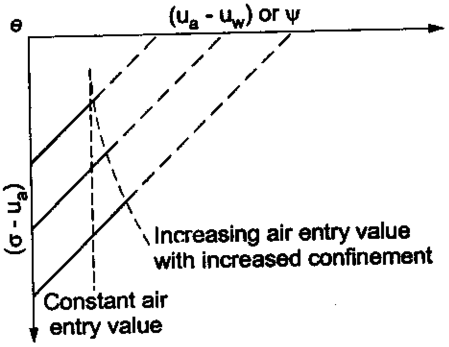

Figure 8. Effect of a variation in the air entry value on the void ratio contours.

The air entry value would be anticipated to increase with a decrease in void ratio. This means that the 45 degree contour would be adhered to for a greater distance from the net total stress reference plane. No attempt is made at this time to define the air entry value of the soil as a function of void ratio (or stress state).

Postulate 6: A gradual curve forms from the air entry value to the secondary reference condition, corresponding to a particular void ratio on the soil structure constitutive surface (Fig. 9).

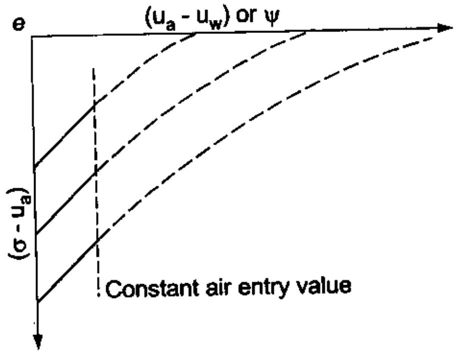

Figure 9. Variation in the constant void ratio contours as the soil becomes increasingly desaturated.

The curve must be tangent at the air entry value and increase in curvature as the secondary reference condition is approached. This means that it should always be possible to join the secondary reference curve provided it is positioned further from void ratio than is the primary reference curve. In other words, at a particular void ratio, the soil suction value should exceed the net total stress value.

The curves should always bend in the direction of the soil suction axis for a clayey, stable-structured soil. For a sandy soil, the curves will bend even more rapidly and may never reach the reference soil suction axis.

The loading portion of the void ratio constitutive surface can be approximated using the steps outlined above. The general character of the void ratio constitutive surface should apply for sands, silts and clays. The greatest difficulty should be observed in defining the constitutive surface near to initial conditions. This is due to the fact that not all of the tests are started from precisely the same stress state and volume-mass state. As well, different tests may follow different stress paths particularly near the start of the test. It is therefore necessary to take into consideration the initial state of the soil. For example, the soil could be initially slurried, compacted or be in an undisturbed state.

The above postulates do not cover all aspects of defining the void ratio constitutive surface. The postulates pertain to the loading (by net total stress or soil suction) constitutive surface of an initially saturated soil. Other postulates are required to define the unloading void ratio constitutive surface. Still other postulates are required for the case where one state variable is increased while the other one may be decreased. The scope of this paper is limited to monotonic loading of an initially saturated soil.

A series of similar postulates are required for the water content constitutive surface. The postulates for the water content surface are outside the scope of this research paper.

4. SUMMARY

A series of postulates have been provided for the estimation of the volume change constitutive surfaces for an unsaturated soil (for the case of monotonic loading). Once again, the estimation procedures make use of the saturated volume change properties of the soil along with the soil-water characteristic curve information. The suggested postulates need to be tested using experimental data on volume change.

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