Finite Element Groundwater Seepage Code Verification

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ABSTRACT: Seepage modeling remains a crucial numerical analysis commonly performed by geotechnical engineers. The needs of the industry have required on-going research in the area of seepage modeling. Comprehensive solution of groundwater seepage has required the evaluation of multiple solvers for the seepage equation. Such multiple solvers require extensive benchmarking efforts to prove performance both in terms of accuracy and speed of solution. Benchmarking must proceed in terms of steady-state as well as transient problems in 1-D, 2-D, and 3-D. Mesh density and solution time in transient seepage models must be optimized. This paper examines the ability of SVFLUX GT to solve a suite of benchmarks for saturated/unsaturated groundwater seepage numerical modeling.

1 INTRODUCTION

Seepage modeling remains a crucial numerical analysis commonly performed by geotechnical engineers. Comprehensive solution of groundwater seepage has required the evaluation of multiple solvers of the seepage equation. Before using any code that seems applicable to the problem at hand, the user must verify it and control the quality of its results by significant tests (Chapuis 1995; Rowe and Nadarajah 1996). Chapuis et al. (2001) provide a consistent framework to verify, calibrate and document the results of a groundwater numerical code from a user's viewpoint, but there is no standard procedure for the verification of a finite element groundwater seepage code.

The verification of a groundwater seepage modeling code should start with simple problems and more progressively towards problems of increased complexity. Benchmarking must proceed in following terms:

- 1-D, 2-D, and 3-D
- Steady-state and transient conditions
- Saturated only, unsaturated only, from saturated to unsaturated, and from unsaturated to saturated flows

This paper examines a suite of benchmarks for groundwater seepage numerical modeling using a newly developed commercial solver. These benchmarks cover all the conditions mentioned above and verify the capability of the software in simulating groundwater seepage problems.

2 BENCHMARKS

2.1 *Case 1: one-dimensional transient unsaturated only flow*

Celia (1990) presented an infiltration example comparing finite difference and finite element solutions. The example represents an approximate description of a field site in New Mexico. The model involved unsaturated infiltration into a column of 100 cm in depth. The model was duplicated in the software package. Material properties presented in the paper were converted from a functional to a digital representation. The results of the software as compared to the finite element results presented by Celia (1990) are shown in Figure 1. The software results indicate correct solution of the infiltration model. The results validate the capacity of the software in simulating unsaturated only flow.

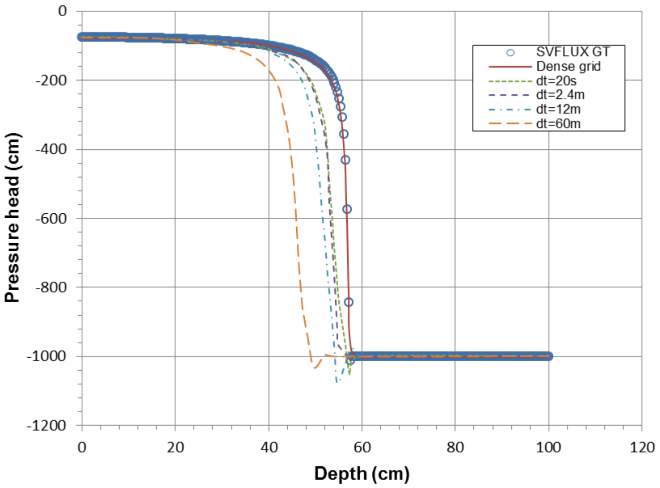


Figure 1 Difference between finite element solutions presented by Celia (1990) and the solution obtained using the software

2.2 *Case 2: two-dimensional steady-state saturated only flow*

The model in this case illustrates a two-dimensional steady-state confined flow under a dam and can be used as the verification for saturated only flow. The material is viewed as saturated with the saturated volumetric water content of 0.4 and a constant saturated hydraulic conductivity of 1e-5 m/s. The dam has two 10 m sheet piles driven partially into the granular soil layer as shown in Figure 2 (Holtz and Kovacs, 1981). On the left side of the dam, the boundary condition is set as constant pressure head (12 m), and on the right side, the boundary condition is assumed as constant pressure head (0 m).

Figure 3 shows the contour of total head (*h*) and several select streamlines under the dam. The distributions of pressure heads at the bottom of the dam (from A to F) are compared between the analytical results and the software in Figure 4. Holtz and Kovacs (1981) noted that this distribution is important for the analysis of the stability of concrete gravity dams. The good agreement between results from the analytical calculation and the software verifies the capability of the software for simulating the saturated only flow.

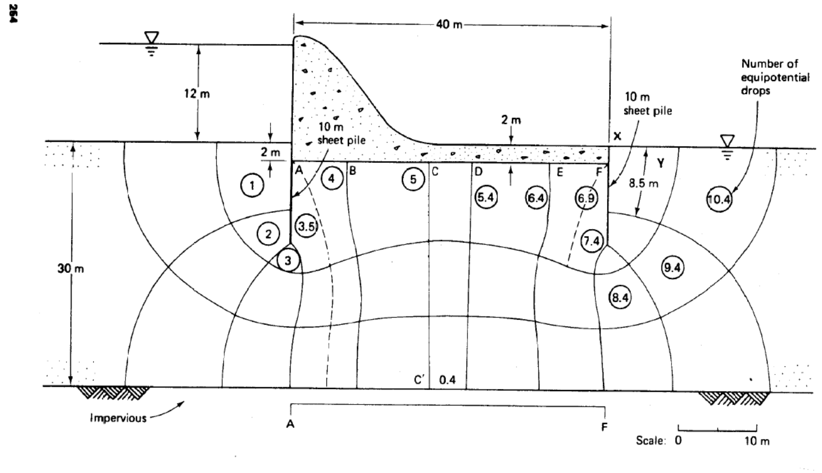


Fig. 7.17 Example of a reasonably well-drawn flow net for confined flow.

Figure 2 Description of the Case 2 example model (Holtz and Kovacs, 1981)

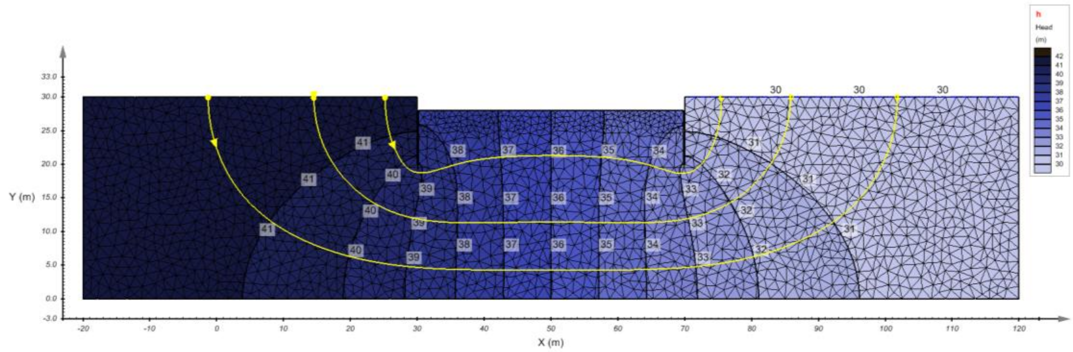


Figure 3 The contours of total head (*h*) and select streamlines under the dam

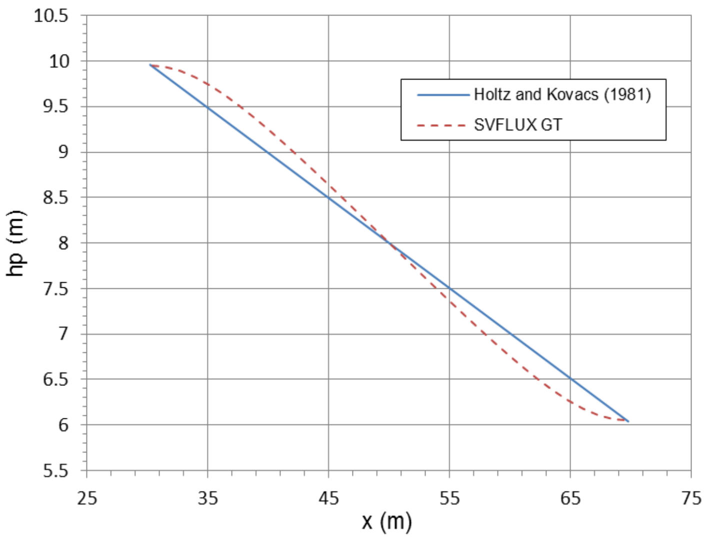


Figure 4 Comparison of the pressure head (*hp*) distributions at the bottom of the dam (from A to F) between the analytical calculation by Holtz and Kovacs (1981) and SVFLUX GT

2.3 *Case 3: two-dimensional steady-state saturated-unsaturated flow*

This verification model considers a vertical cross-section of an unconfined groundwater seepage system in a homogeneous earth dam underlain by an impervious base, and a free-surface and a seepage face appear atop the flow region as shown in Figure 5. The geometry of the simulation domain is a 20 m × 20 m square, and an initial water table line is set at the top surface. The boundary condition on the left side is assigned as Head Constant = 20 m. On the right side, the boundary condition from y = 0 m to y = H_0 is assigned as Head Constant = H_0, and from y = H_0 to y = 20 m as Review Boundary Condition to determine the length of the seepage face. The material is viewed as unsaturated with the saturated hydraulic conductivity of 3.5 m/s. The Fredlund and Xing Equation (1994) is used for fitting the soil-water characteristic curve, and the modified Campbell Estimation (1974) is used for unsaturated hydraulic conductivity estimation. Figure 6 shows the contours of pore-water pressure, flow field and the final location of water table line for the case of H_0 = 6 m obtained from the software. The seepage face length results from the software and the paper by Lee and Leap (1997) are summarized in Table 1. From the comparisons it can be seen that the results from the software are close to the simulation results from Lee and Leap (1997). Some differences exist because the length of the seepage face is very sensitive to the mesh density along the boundary near the exit point and in the area nearby. This verification example verifies the capability of the software in handling the simulation of saturated-unsaturated seepage flow, free surface and seepage face in steady state.

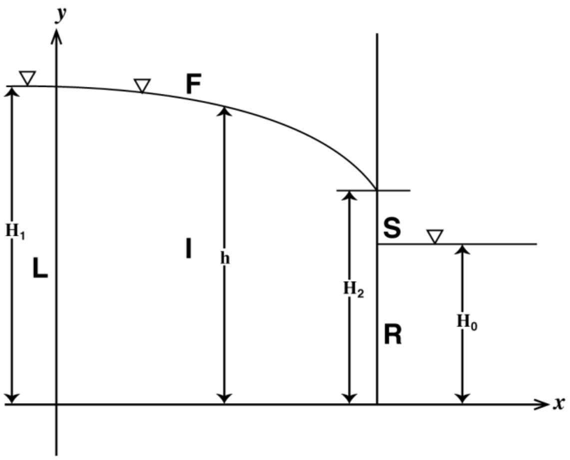


Figure 5 Physical domain of the Case 3 model showing free surface (F) and seepage face (S)

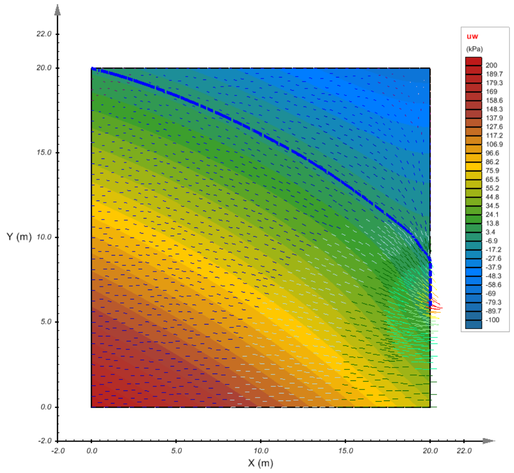


Figure 6 The contours of pore-water pressure, flow field and the final location of the water table line for the case of H_0 = 6 m

Table 1 Seepage face length results and comparisons of Case 3 models

H_1 (m)	H_0 (m)	S (m)* (Lee, 1997)	S (m) (analytical)	S (m) (SVFLUX GT)	Error**
20	2	5.7	5.3	5.76	1.0%
20	4	4.0	3.7	4.10	2.5%
20	6	2.7	2.4	2.58	4.4%
20	8	1.7	1.5	1.80	5.5%
20	10	0.9	0.8	1.00	11.1%

*Simulation results from Lee and Leap, 1997.

**Compare with the simulation results from Lee and Leap, 1997.

2.4 *Case 4: two-dimensional transient unsaturated-saturated flow*

This verification model illustrates unsteady-state groundwater seepage below a lagoon. The lagoon is placed on top of a 1 m thick soil linear, and the total height of the model is 10 m as shown in Figure 7. The geometry of the problem is symmetrical, and the liner and the surrounding soil are assumed to be isotropic with respect to their hydraulic conductivity.

An initial condition with a water table located 5 m below the ground surface is assumed. On the right boundary, a constant head (5 m) boundary condition is set below the water table, and the other surfaces are assumed as "Zero Flux". The lagoon is set as a constant pressure head (1 m) boundary condition to assume that the lagoon is filled with water to 1 m height at the time being equal to 0. The materials are viewed as unsaturated, and the saturated hydraulic conductivities of surrounding soil and soil liner are 0.036 m/hr and 0.018 m/hr, respectively. More detailed descriptions about unsaturated material properties can be found in the book by Fredlund and Rahardjo (1993).

In this transient model, the solution is run for 200 hours, and it can be seen to reach the steady state at 189 hours according to Fredlund and Rahardjo (1993). Figure 8, Figure 9 and Figure 10 show the comparisons of pressure head contours from Fredlund and Rahardjo (1993) and the software at the times of 7 hours, 13 hours and steady state. The results from the software are in good agreement with those from Fredlund and Rahardjo (1993). This example further verifies the capability of the software for unsaturated-saturated seepage simulations in transient state.

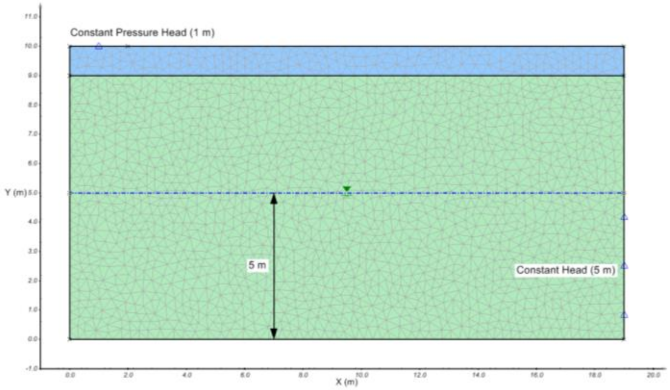
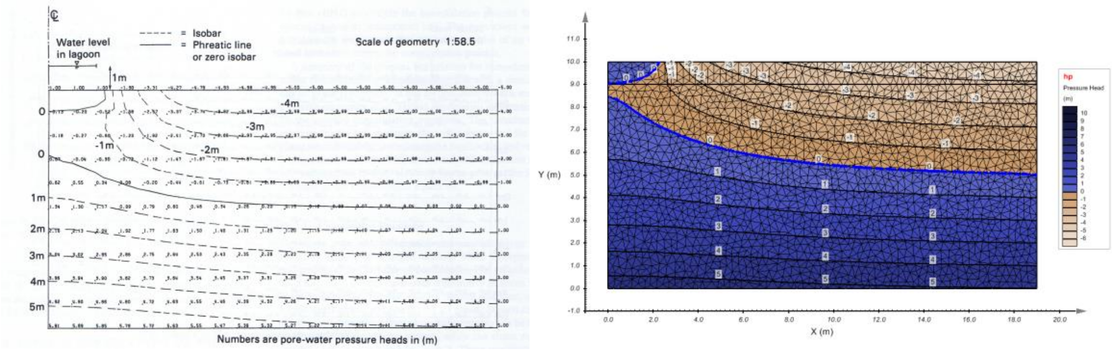
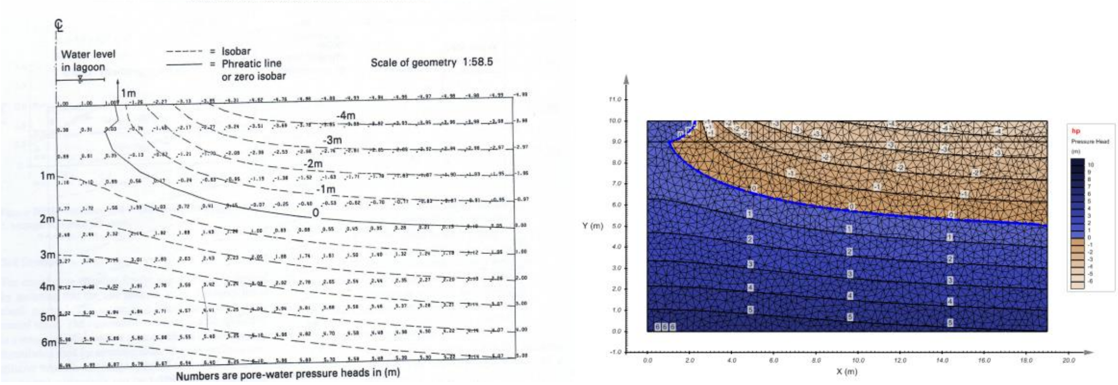


Figure 7 Geometry and boundary conditions of the Case 4 model



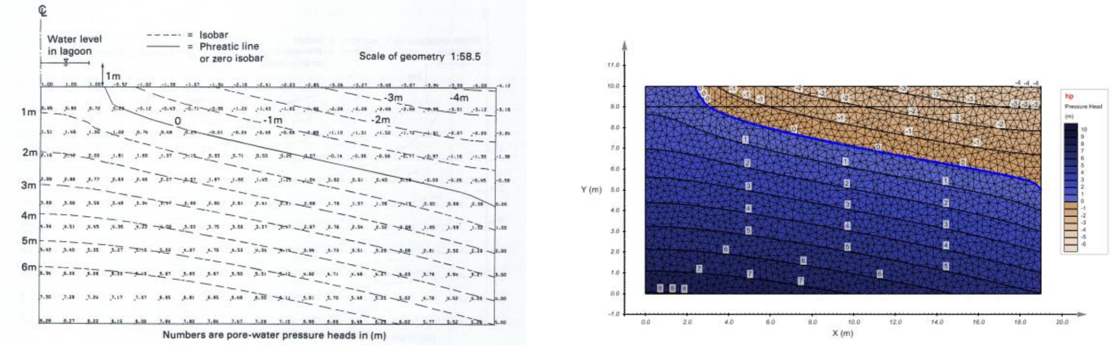
(a) (b)

Figure 8 The comparison of pressure head contours from (a) Fredlund and Rahardjo (1993) and (b) the software at 7 hours.



(a) (b)

Figure 9 The comparison of pressure head contours from (a) Fredlund and Rahardjo (1993) and (b) the software at 13 hours.



(a) (b)

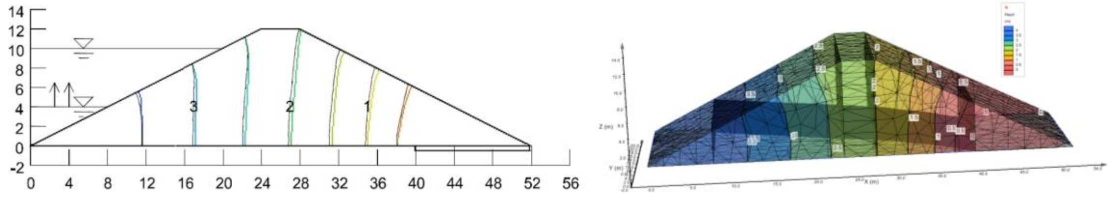
Figure 10 The comparison of pressure head contours from (a) Fredlund and Rahardjo (1993) and (b) the software at the steady state.

2.5 *Case 5: three-dimensional steady-state and transient unsaturated flow*

This three-dimensional seepage verification model considers the transient seepage through an earth dam in the situation of the rapid filling of a reservoir. The 3D geometry of the earth fill dam is 12 m high, 52 m in length and extruded from the 2D model with a width of 20 m. The initial conditions of head were obtained by first solving a steady-state run of the model with the head on the upstream face of the dam set to 4 m and a head of 0 m on the lower portion of the filter. The result from the steady-state analysis is then imported as the initial conditions for the transient analysis and compared with the 2D contours from Pentland (2001) as shown in Figure 11.

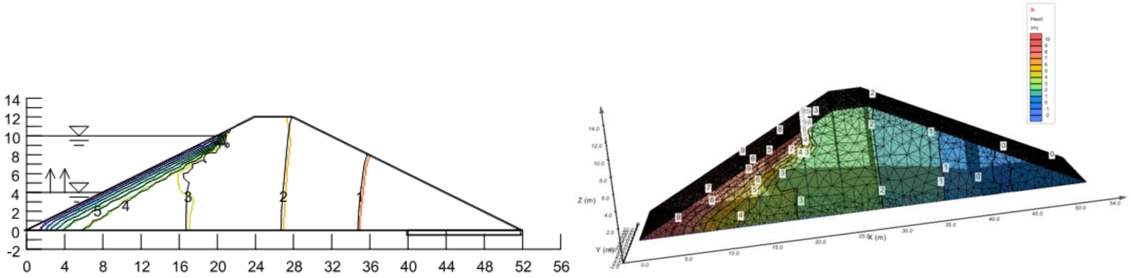
The material properties are considered as unsaturated and remain the same in the transient flow model. The boundary conditions change slightly. A head of 10 m is set on the upstream face of the dam to simulate a full reservoir condition. The model is run for 16,383 hours. Below, Figure 12 and Figure 16 show the head contours from Pentland (2001) and the software at times of 15 and 16,383 hours, and more results of 3D total head contours at times of 15, 225, 1,023, 4,095 and 16,383 hours from the software are also provided as shown in Figure 13, Figure 14 and Figure 15.

It can be seen from the above figures that the results computed by the software are in good agreement with those from Pentland (2001). This model further verifies the ability of the software for simulating the groundwater seepage flow in three-dimensional problems.



(a) (b)

Figure 11 Head contours at the initial condition from (a) Pentland (2001) and (b) the software



(a) (b)

Figure 12 Computed head contours at time 15 hours from (a) Pentland (2001) and (b) the software

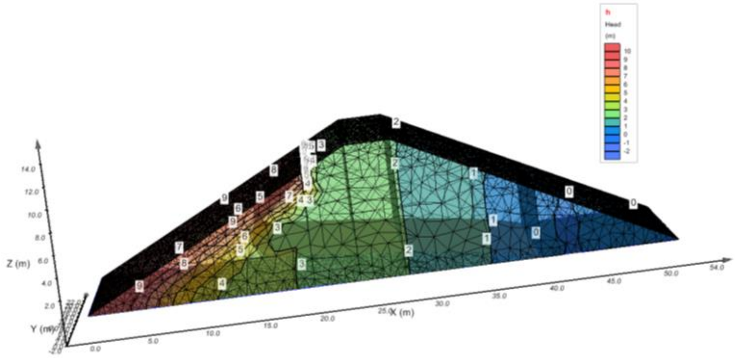


Figure 13 3D head contours at time 225 hours from the software

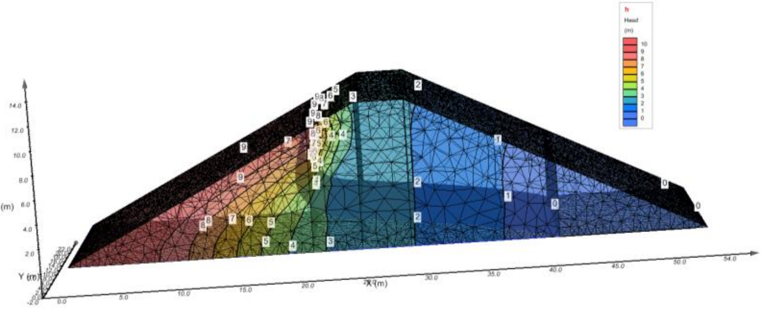


Figure 14 3D head contours at time 1,023 hours from the software

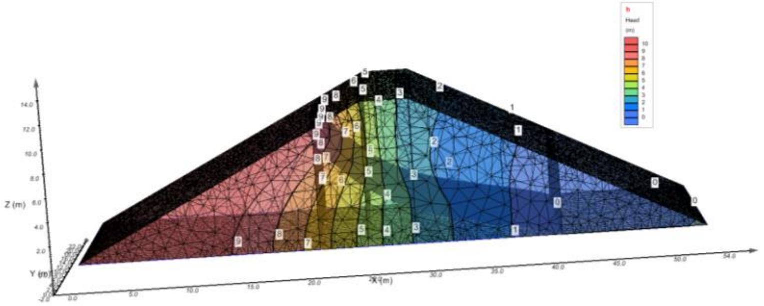
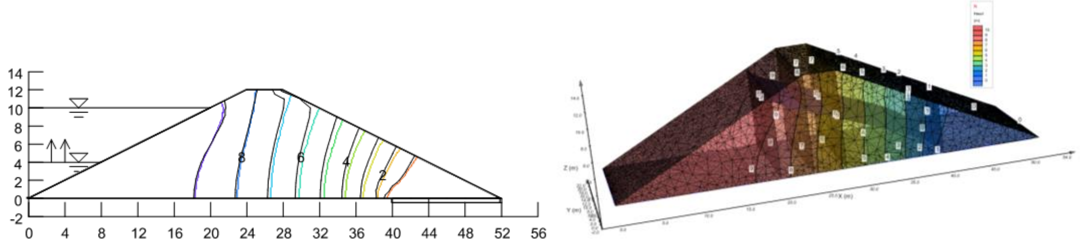


Figure 15 3D head contours at time 4,095 hours the software



(a) (b)

Figure 16 Computed head contours at time 16,383 hours from (a) Pentland (2001) and (b) the software

3 CONCLUSIONS

Seepage modeling remains a crucial numerical analysis commonly performed by geotechnical engineers. Comprehensive solution of groundwater seepage has required the verification of the seepage equation. Benchmarking must proceed in terms of steady-state as well as transient problems in 1-D, 2-D, and 3-D. This paper provides a comprehensive verification for finite element groundwater seepage code using the software. Several example models are successfully verified, and these verification examples cover almost all different types of seepage problems, which include:

- one-dimensional, two-dimensional and three-dimensional problems
- steady-state and unsteady-state conditions
- saturated only, unsaturated only, from saturated to unsaturated and from unsaturated to saturated flows

Currently, there is no standard procedure for the verification of commercial finite element groundwater seepage codes. For any commercial finite element groundwater seepage code, verifications should be provided as presented in the current paper. It must be noted that correctly solving verification examples does not fully guarantee that a groundwater code is always valid for any type of problem. Ongoing development and testing in the area of seepage modeling is required. These successful benchmarks verify the capability of the software in solving groundwater seepage problems.

4 REFERENCES

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