A Methodology For Applying Probability Theory To Unsaturated Hydraulic Properties as the Foundation for Seepage Analysis

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ABSTRACT: Methods for application of unsaturated soils analysis have progressed greatly in the past few years. The consideration of unsaturated scenarios in the numerical modeling of seepage in soils as well as the influence on slope stability is becoming more commonplace in geotechnical consulting firms. Given that the dissipation of soil suction is often a trigger mechanism in slope failures it is becoming increasingly important to apply probability theory to the determination of reasonable flow scenarios in soil slopes. This is especially true in light of the reasonable variation in climatic influences. This paper intends to describe a reasonable methodology for the describing of unsaturated hydraulic properties of a soil for application in a numerical modeling seepage analysis.

INTRODUCTION

A great deal of research effort has been placed on describing the shape and character of unsaturated hydraulic soil properties over the past few decades. Early work on the soil-water characteristic curve (SWCC) focused on the mathematical representation (Brooks and Corey, 1964; Gardner, 1956; van Genuchten, 1980; Fredlund & Xing, 1994, Gitirana, 2004, and others). The majority of these mathematical representation methods sought to represent the SWCC with increasingly complex mathematical equations which more accurately described the unsaturated hydraulic behavior of a wide variety of soil textures and types. The early development paralleled the computational developments on-going in the world. For example, the focus on early equations such as Brooks and Corey (2 parameter) and Gardner (2 parameter) equations was simplicity and minimizing the number of parameters such that the application of these equations to practice could be simplified.

After the introduction of the PC by IBM in 1981 the calculation of points using these equations as well as the application of non-linear least-squared regression algorithms to the fitting process minimized the necessity to reduce the number of equation parameters. This led to the popularization of equations with three and four parameters such as van Genuchten and Fredlund and Xing. Use of additional equation parameters allowed the equations to fit a larger variety of soil types as well as more accurately represent the behavior of dry soils with suctions far past the wilting point of 1500 kPa (Fredlund, 1993) The application of these fitting equations may be seen in seepage finite element packages such as SVFlux (SoilVision Systems Ltd., 2007).

The primary difficulty in applying existing unsaturated hydraulic functions is that the parameters used in the equations are inter-dependant and therefore, non-unique. For example, the Fredlund and Xing a_f parameter affects both the n_f and m_f parameters and varying one will influence the other. It is therefore impossible to provide a reasonable statistical distribution of a SWCC equational parameter from a given dataset which is unique.This difficulty was recognized in a statistical study of seepage and slope stability (Gitirana, 2004) and a new equation for representing the SWCC was developed. This new equation had the added benefit in that the equation parameters were independant of each other. Therefore the parameters could be statistically described.

This paper focuses on a general methodology of the application of unsaturated hydraulic parameters such that any legacy equations previously used in the representation of the SWCC or the unsaturated hydraulic conductivity. These curves may then be used as the basis for a probability theory analysis.

SWCC REPRESENTATION

The soil-water characteristic curve (SWCC) or soil-moisture retention curve (as it is termed in the field of Soil Science) has traditionally been characterized by the equation parameters of the fitting equation. Therefore a group of soils may be described by the Brooks and Corey α and λ parameters or the Fredlund & Xing a_f, n_f, and m_f parameters. Although the a_f parameter is loosely related to the air-entry value, it is also influenced by other equation parameters and is therefore not unique. Central tendency theory therefore does not apply to these equation parameters in the sense that a specific a_f parameter corresponds to a specific air-entry value.

An improved methodology characterizes the SWCC by the physical properties of soil such as the porosity, n, the air entry value (ψ_{ae}), the average slope of the drying curve, the residual water content, θ_r, and the residual suction, ψ_r. As the slope of the drying curve can be determined based on the porosity, AEV, residual suction and water content we will not consider it as a parameter in this methodology. These parameters and their influence on the shape of the SWCC may be seen in FIG. 1.

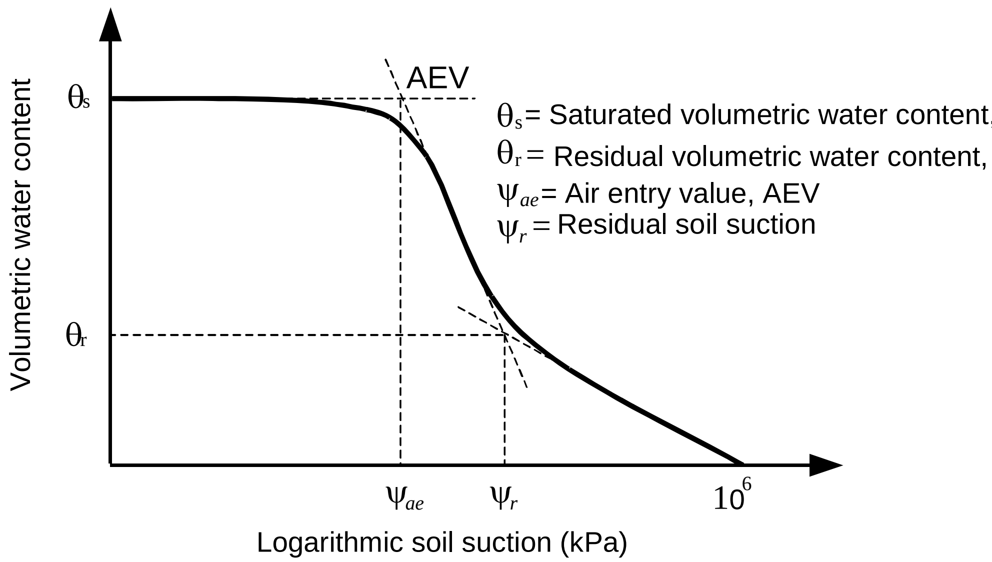

FIG. 1. Conceptual model of physically realistic parameters to represent the soil-water characteristic curve.

These characterization parameters may then be determined in one of two ways; i) through fitting equation parameters of the Gitirana (2004) equation, or, ii) by using a construction technique as outlined by Vanapalli and Fredlund (1998). This paper will focus on the use of the construction technique as it may be applied independant of the SWCC fitting method.

UNSATURATED HYDRAULIC CONDUCTIVITY REPRESENTATION

Representation of the unsaturated hydraulic conductivity curve may be idealized by three lines; i) a horizontal line over the saturated range, ii) a sloped line over primary soil desaturation, and, iii) a residual constant hydraulic conductivity at which vapor diffusion is the governing process which may reasonably be assumed to be between 1e-10 m/s and 1e-13 m/s (Ebrahimi-Birang et al., 2004). Although a number of functional relationships have been presented which relate the SWCC to the unsaturated hydraulic conductivity curve (van Genuchten, 1980; Fredlund & Xing, 1994; Brooks and Corey, 1964, as well as others), they all follow a similar basic formula. The air-entry value, $\psi_{bk,}$ is commonly considered to be the same between the SWCC and the unsaturated hydraulic conductivity curve and there is an assumption regarding the mapping of the desaturation of the soil with the resulting reduction in hydraulic conductivity.

The η parameter may be defined as the slope of the unsaturated hydraulic conductivity curve on a log-log plot. This parameter corresponds directly to the n_g slope parameter used in the Gardner equation.

Representing the hydraulic conductivity curve with these parameters ultimately allows the curve to be represented by parameters which are i) physically meaningful and ii) independent of a particular fitting or estimating method.

STATISTICAL METHODOLOGY WITH LIMITED DATA

A common difficulty with application to specific site analysis is the presence of data gaps. A typical site characterization program may consist of the following set of typical data.

Sieve analysis	x 150 samples	Specific gravity	x 22 samples
Density	x 150 samples	SWCC	x 7 samples
Water contents	x 150 samples	ksat	x 25 samples

The data above might be a typical representation of the data passed to a numerical modeling program. It is desired to perform a seepage analysis based on the above data. The following process presents a reasonable methodology for extraction of the utmost information from this dataset.

Sieve and hydrometer information represent valuable and often underutilized pieces of information. Inherent in a particle-size distribution is a statistical probability density function (PDF) indicating the frequency of each particle size. A significant amount of research has been performed in order to translate the particle-size distribution to an equivalent pore-size distribution (Harr, 1977). This type of translation is commonly referred to as a pedo-transfer function (PTF) in the Soil Science profession.

The common difficulty with this type of function is that the results of the estimation are influenced by structure, compression history, mineralogy and other such parameters which are often somewhat site-specific. It is therefore considered ideal to calibrate these estimations (pedo-transfer functions) to a specific site. Once a pedo-transfer function has been calibrated to a certain site the confidence of the estimations is significantly increased. Given the data outlined previously it is therefore possible to proceed in the following manner with the development of unsaturated hydraulic soil properties:

i) Mathematically fit the grain-size distributions (and hydrometer data) with a mathematical equation,
ii) Calibrate a pedo-transfer function with existing Tempe cell SWCC data to develop a reasonable understanding of the packing porosity of a soil.
iii) Use the calibrated pedo-transfer function to predict SWCC curves for the remaining grain-size distributions
iv) Estimate the unsaturated hydraulic soil properties using the measured ksat information.

The benefit of such an approach is that it provides a calibrated mapping function between the SWCC and the grain-size distribution. This mapping function allows the estimation of 150 SWCCs and unsaturated hydraulic conductivity curves for the theoretical site based on the original seven Tempe cell tests. This data then forms the basis for reducing the soil behavior to equation-independent unsaturated parameters which may then be used as the basis for a probabilistic seepage analysis.

SAMPLE APPLICATION

Work at a typical site might involve the geotechnical testing described in the above scenario. The data quantities obtained from a field characterization program are similar to those presented in the previous section.

The grain-size distributions are fit with the Fredlund (2000) grain-size equation. Then all grain-size distributions are mapped to an equivalent pore-size distribution (SWCC) using a pedo-transfer function . In this case the Fredlund & Wilson (2000) pedo-transfer function is used. The Fredlund & Wilson (2002) pedo-transfer function makes use of a *Packing Porosity* variable which generally accounts for uncertainties in the estimation method. The *Packing Porosity* is determined in software through the use of a neural net algorithm.

The estimated SWCCs are then compared to the seven measured SWCCs and there may be a difference between the average of all estimated SWCCs and the measured SWCCs. It is then assumed that the average of all measured and estimated SWCCs must be the same since the data is taken from similar materials the same site. The difference between estimated and measured SWCCs is assumed to be accounted for in the *Packing Porosity* variable and the global estimate of the *Packing Porosity* is shifted until the averages between the measured and estimated SWCCs match.. This is the “training” step.

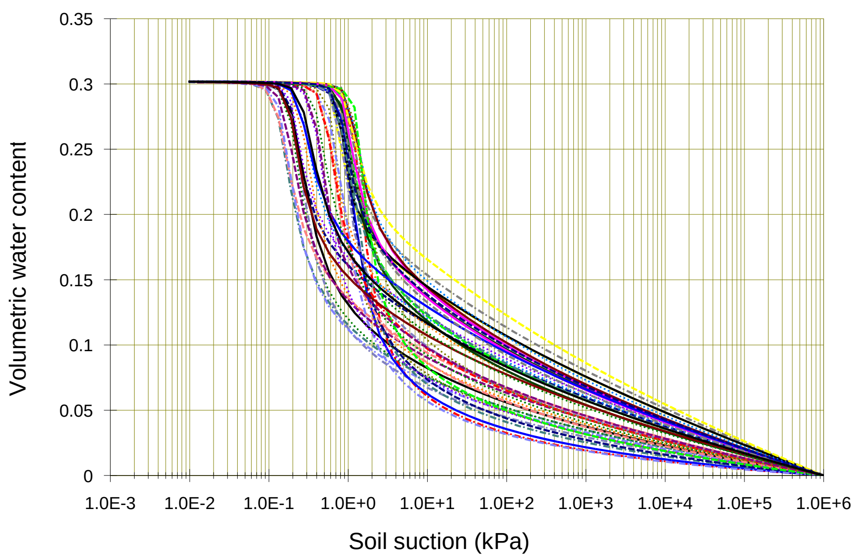

FIG. 2. Group of theoretically estimated soil-water characteristic curves based on grain-size distributions

Once the SWCC estimation function has been “trained” it may be used to estimate a variety of SWCC curves. Each of these curves can be reduced to the standardized description parameters as outlined in previous chapters to provide a mean and standard deviation on the porosity, n, the air entry value (ψ_b), the average slope of the drying curve, the residual water content, θ_r, and the residual suction, ψ_r. In this particular case the distribution of theoretical SWCCs was obtained as shown in FIG. 2. The physical parameters for each SWCC may then be determined using the construction technique. This results in a mean and standard deviation for each parameter.

What is of primary interest in obtaining unsaturated hydraulic conductivity curves is obtaining i) the air-entry value and ii) the slope of the unsaturated hydraulic conductivity curve. It is reasonable to assume that the air entry value (AEV) will be the same.

The slope, η, of the unsaturated hydraulic curve becomes the primary unknown to determine. Since not just a single value is desired but a justifiable distribution there is need of a methodology in which a suitable group of curves can be determined based on the existing grain-size distributions and the theoretically developed group of soil-water characteristic curves.

The historical approach is to use an existing pedo-transfer function which translates between the SWCC and the unsaturated hydraulic conductivity curve (Fredlund & Xing, 1994; van Genuchten, 1981). With this approach the constant slope, η, of the curve may be approximated based on an existing pedo-transfer function.

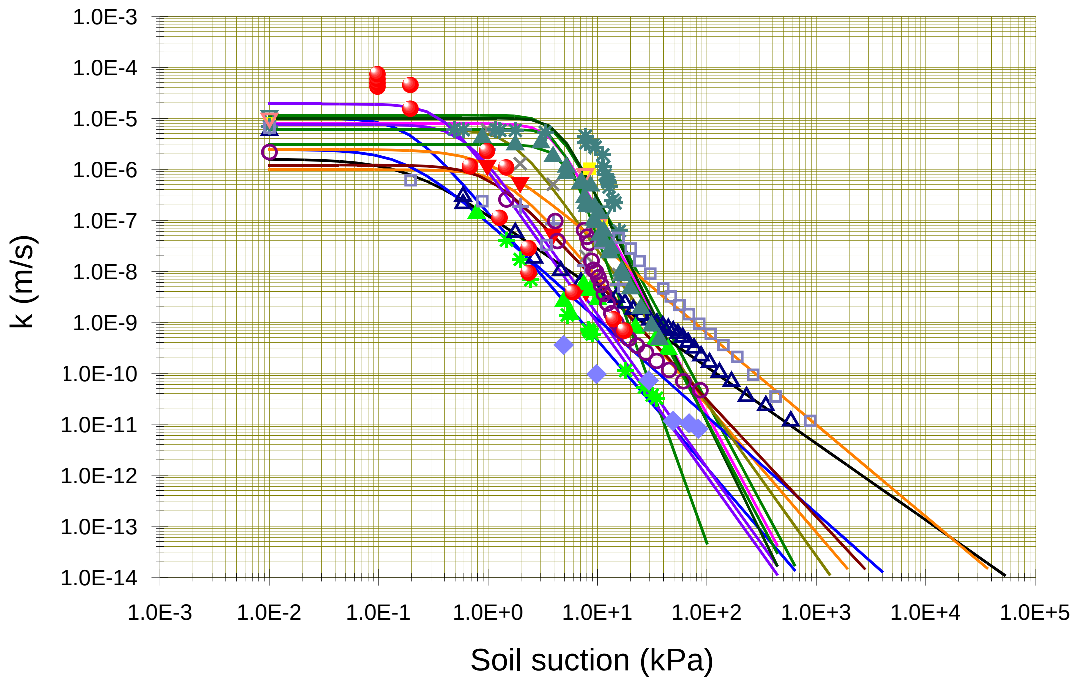

FIG. 3. Group of hydraulic conductivity unsaturated curves matching the grain-size distributions.

Another methodology which may be utilized is to compare to curves already present in the SoilVision® dataset for an indication of the reasonable variation of the η parameter. The comparison is based on matching grain-size distributions to measured values and plotting corresponding unsaturated hydraulic conductivity curves measured in the laboratory. The high cost of measuring unsaturated hydraulic conductivity curves means that obtaining such data is often cost-prohibitive.

A group of typical unsaturated hydraulic conductivity curves selected from the SoilVision® database using this methodology may be seen in Figure 5.

At this point in the methodology a mean and standard deviation has been determined for all primary, physically –based parameters in order to perform an unsaturated analysis. The end result is statistically based seepage modeling to which probability theory may be applied. This can result in relative estimates of flow as shown in the following tornado diagram. The reasonable variation in the 1D vertical flow model can be determined with reasonable certainty and the influence of model input parameters can be identified.

An example of how the output of this type of analysis may be presented may be seen in FIG. 4. In this particular figure the influence of the various parameters in the 1D model is summarized. The methodology to generate this figure is the point estimation technique (PEM) and this analysis method provides two valuable pieces of information i) which parameters have the opportunity to most significantly influence the output, and, the potential and reasonable impact of these parameters.

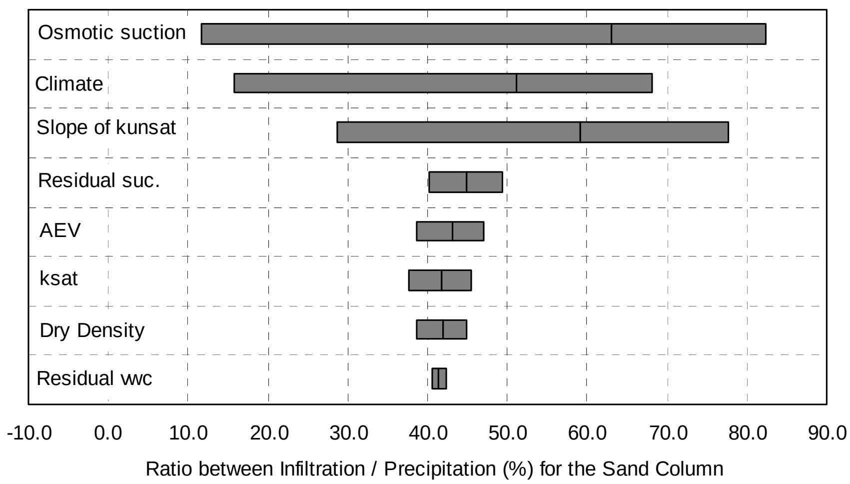

FIG. 4. Tornado diagram illustrating the influence of various soil input parameters on infiltration.

CONCLUSIONS

This paper presents a methodology for the quantification of unsaturated soils for the purpose of applying probability theory to a seepage analysis. The methodology is designed to be independent of any particular fitting or estimation method and may be applied to earth cover design, flow in waste rock, flow in earth dams, as well as numerous other applications. The application of this methodology illustrates its value in the practice of geotechnical engineering.

REFERENCES

Brooks, R.H., and A.T. Corey. (1964). "Hydraulic properties of porous media." Hydrol. Pap. 3. Colorado State Univ., Fort Collins.

Ebrahimi-Birang, N., Gitirana Jr., G. F. N., Fredlund, D. G., Fredlund, M. D., and Samarasekera, L., (2004). "A lower limit for the water permeability coefficient." Proceedings of the 57th Canadian Geotechnical Conference, Quebec City. Oct. 24-27. pp.12-19

Fredlund D.G. & Rahardjo H., (1993). Soil Mechanics for Unsaturated Soils, John Wiley & Sons, Inc.

Fredlund, D.G., and Xing A. (1994). "Equations for the soil-water characteristic curve." Canadian Geotechnical Journal, 31: 521–532.

Fredlund, D.G., Xing, A., and Huang, S., (1994). "Predicting the permeability function for unsaturated soil using the soil-water characteristic curve." Canadian Geotechnical Journal, Vol. 31, No. 3, pp. 533-546.

Fredlund, M.D. (2000). The Role of Unsaturated Soil Property Functions in the Practice of Unsaturated Soil Mechanics, University of Saskatchewan, pp.292

Fredlund, M.D.,. Wilson, G.W & Fredlund, D.G., (2000). An Equation to Represent grain-size Distribution, Canadian Geotechnical Journal, v37, 817-827.

Fredlund, M.D., Fredlund, D.G. & Wilson, G.W., (2002). Use of the grain-size distribution for the estimation of the soil-water characteristic curve, Canadian Geotechnical Journal, Vol. 39, No.5, pp.1103-1117

Gardner, W. (1956). "Mathematics of isothermal water conduction in unsaturated soils." Highway Research Board Special Report 40 International Symposium on Physico-Chemical Phenomenon in Soils, Washington D.C. pp 78-87.

Gitirana, G.F.N., Fredlund, D.G., (2004). "Soil-Water Characteristic Curve Equation with Independent Properties." Journal of Geotechnical and Geoenvironmental Engineering, ASCE, February, pp. 209-212.

Gitirana, G.F.N., (2005). "Weather-Related Geo-Hazard Assessment Model for Railway Embankment Stability." Ph.D. Thesis, University of Saskatchewan, Saskatoon, Saskatchewan, Canada, 411p.

Harr, M.E., (1977). "Mechanics of Particulate Media – A Probabilistic Approach." McGraw-Hill International Book Company, New York.

SoilVision Systems Ltd., (2007). SVFlux User's Manual, SVOffice 2006, Saskatoon, SK, Canada

Vanapalli S.K., Sillers W.S., and Fredlund M.D., (1998). "The Meaning And Relevance Of Residual State To Unsaturated Soils." 51st Canadian Geotechnical Conference, Edmonton, Alberta, Canada, October 4-8

van Genuchten, M.Th. (1980). "A closed-form equation for predicting the hydraulic conductivity of unsaturated soils." Soil Sci. Soc. Am. J. 44:892-898.