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Abstract

The measurement of soil parameters, such as the permeability and shear strength functions, used to describe unsaturated soil behaviour can be expensive, difficult and often impractical to obtain. This paper proposes a model for predicting the shear strength (versus matric suction) function of unsaturated soils. The prediction model uses the soil-water characteristic curve and the shear strength parameters of the saturated soil (i.e., effective cohesion and effective angle of internal friction). Once a reasonable estimate of the soil-water characteristic curve is obtained, satisfactory predictions of the shear strength function can be made for the unsaturated soil. Closed-form solutions for the shear strength function of unsaturated soils are obtained for cases where a simple soil-water characteristic equation is used in the prediction model.

Key words : Soil suction, soil-water characteristic curve, shear strength function, unsaturated soil

Introduction

A theoretical framework for unsaturated soil mechanics has been firmly established over the past couple of decades. The constitutive equations for volume change, shear strength and flow for unsaturated soil have become generally accepted in geotechnical engineering (Fredlund and Rahardjo, 1993a). The measurement of soil parameters for the unsaturated soil constitutive models, however, remains a demanding laboratory process. For most practical problems, it has been found that approximate soil properties are adequate for most analyses (Fredlund, 1995). Hence, empirical procedures to estimate unsaturated soil functions are adequate.

Laboratory studies have shown that there is a relationship between the soil-water characteristic curve and the unsaturated soil properties (Fredlund and Rahardjo, 1993b). Several models have been proposed to empirically predict the permeability function for an unsaturated soil from the soil-water characteristic curve by using the saturated coefficient of permeability as the starting value (Fredlund et al, 1994). This paper provides engineers with a means of estimating the shear strength function for an unsaturated soil from the soil-water characteristic curve by using the saturated shear strength parameters as the starting values.

Literature Review

The shear strength of a soil is required for numerous analysis such as; the prediction of the stability of slopes, the design of foundations and earth retaining structures. The effective stress variable proposed by Terzaghi (1936) has been used in the Mohr-Coulomb theory for predicting the shear strength of *saturated* soils. The shear strength equation for saturated soils is expressed as a linear function of effective stress and is given as follows.

$$\tau = c' + (\sigma_n - u_w)\tan\phi' \qquad [1]$$

where :
- τ = shear strength
- c' = effective cohesion,
- ϕ' = effective angle of internal friction,
- σ_n = total normal stress on the plane of failure, and
- $(\sigma_n - u_w)$ = effective normal stress on the plane of failure.
- u_w = pore-water pressure.

Many practical problems involve assessing the shear strength of *unsaturated* soils. Fredlund and Morgenstern (1977) showed that the shear strength of unsaturated soils can be described by any two of three stress state variables, namely, $(\sigma - u_a)$, $(\sigma - u_w)$, and $(u_a - u_w)$, where u_a is the pore-air pressure. Fredlund et al. (1978) proposed the following equation for the shear strength of unsaturated soils:

$$\tau = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b \qquad [2]$$

where:

ϕ^b = angle indicating the rate of increase in shear strength relative to a change in matric suction, $(u_a - u_w)$, when using $(\sigma_n - u_a)$ and $(u_a - u_w)$ as the two state variables, and

ϕ' = angle indicating the rate of increase in shear strength with respect to net normal stress, $(\sigma_n - u_w)$ when using $(\sigma_n - u_w)$ and $(u_a - u_w)$ as the two state variables.

The effects of changes in total stress and pore-water pressure are handled in an independent manner in Eq. [2]. Equation [2] can be rewritten in the following form:

$$\tau = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\beta\tan\phi' \qquad [3]$$

where: $\beta = \tan\phi^b / \tan\phi'$

Beta, β, represents the decrease in effective stress resistance as matric suction increases. As such, β varies from 1 at saturation to a low value at low water contents. This means that the angle ϕ^b is equal to ϕ' at saturation and then reduces with suction.

Lamborn (1986) proposed a shear strength equation for unsaturated soils by extending a micro-mechanics model based on principles of irreversible thermodynamics to the energy versus volume relationship in a multi-phase material (i.e., solids, fluids and voids). The equation is as follows:

$$\tau = c' + (\sigma - u_a)\tan\phi' + (u_a - u_w)\,\theta_w \tan\phi' \qquad [4]$$

where: θ_w = volumetric water content which is defined as the ratio of the volume of water to the total volume of the soil.

The volumetric water content, θ_w, decreases as matric suction increases and it is a non-linear function of matric suction. However, it should be noted that the friction angle associated with matric suction does not become equal to ϕ' at saturation unless the volumetric water content is equal to one.

For soils having a degree of saturation less than 85 percent, Peterson (1988) proposed the following shear strength equation.

$$\tau = c' + (\sigma - u_a)\tan\phi' + C_\psi \qquad [5]$$

where: C_ψ = apparent cohesion due to suction.

The influence of soil suction on shear strength in Eq. [5] is considered as an increase in the cohesion of the soil. The apparent cohesion due to suction, C_ψ, is dependent on the water content of the soil. Equation [5] is equivalent to Equation [2] when the apparent cohesion, C_ψ, is expressed as being equal to $[(u_a - u_w)\tan\phi^b]$.

Equations for shear strength were also proposed by Satija (1978), Karube (1988) and Toll (1990). Most of the shear strength equations for unsaturated soils in the literature are either linear or bi-linear approximations. A non-linear model is more realistic and should provide a better approximation. While numerous forms have been proposed for the unsaturated shear strength equation, there has been little verification of the equations with experimental data.

Soil-Water Characteristic Curve

The soil-water characteristic curve for a soil is defined as the relationship between water content and suction. The water content variable (i.e., volumetric water content, gravimetric water content or degree of saturation) defines the amount of water contained in the pores of the soil. The variable is often used in a dimensionless form where the water content is referenced to a residual or zero water content.

$$\Theta = \frac{\theta - \theta_r}{\theta_s - \theta_r} \qquad [6]$$

where: θ = volumetric water content at any suction (or $\theta(u_a - u_w)$),
θ_s = volumetric water content at saturation,
θ_r = volumetric water content at residual conditions, and
Θ = normalized volumetric water content. When the reference volumetric water content, θ_r is taken as being zero, Θ is equal to θ/θ_s.

The suction may be either the matric suction, (i.e., ($u_a - u_w$)), or total suction (i.e., matric plus osmotic suction) of the soil. At high suctions (e.g., greater than about 3000 kPa), matric suction and total suction are generally assumed to be essentially the same.

The total suction corresponding to zero water content appears to be essentially the same for all type of soils. A value slightly below 1,000,000 kPa has been experimentally supported for a variety of soils (Croney and Coleman, 1961; Russam and Coleman, 1961; Fredlund, 1964). The value is also supported by thermodynamic considerations (Richards, 1965). In other words, there is a maximum total suction value corresponding to a zero relative humidity in any porous medium. A general equation describing the soil-water characteristic curve over the entire suction range (i.e., 0 to 1,000,000 kPa) has been given by Fredlund and Xing (1994).

$$\theta = \theta_s \left(1 - \frac{\ln(1 + \psi / \psi_r)}{\ln(1 + 1000000 / \psi_r)}\right) \left[\frac{1}{\ln\left(e + (\psi / a)^n\right)}\right]^m \qquad [7]$$

where: ψ = total soil suction (kPa),
e = natural number, 2.71828...,
ψ_r = total suction (expressed in kPa) corresponding to the residual water content, θ_r,
a = a soil parameter which is related to the air entry value of the soil (kPa),
n = a soil parameter which controls the slope at the inflection point in the soil-water characteristic curve,
m = a soil parameter which is related to the residual water content of the soil.

The parameters, a, n and m, in Eq. [7] can be determined using a non-linear regression procedure outlined by Fredlund and Xing (1994). The residual water content, θ_r, is assumed to be zero. The normalized (volumetric or gravimetric) water content when referenced to zero water content is equal to the degree of saturation, S, provided the total volume change is negligible (Fredlund et al, 1994).

The shear strength of a soil is a function of matric suction as it goes from the saturated condition to an unsaturated condition. In turn, the water content is a function of matric suction. Equation [7] can be expressed in terms of the matric suction of the soil:

$$\theta = \theta_s \left(1 - \frac{\ln(1 + (u_a - u_w)/(u_a - u_w)_r)}{\ln(1 + 1000000/(u_a - u_w)_r)}\right)\left[\frac{1}{\ln(e + ((u_a - u_w)/a)^n)}\right]^m \quad [8]$$

where: $(u_a - u_w)_r$ = the matric suction corresponding to the residual water content, θ_r.

A Model for the Shear Strength Function for Unsaturated Soils

The contribution of matric suction to shear strength of an unsaturated soil can be assumed to be proportional to the product of matric suction, $(u_a - u_w)$, and the normalized area of water, a_w, at a particular stress state (Fredlund et al, 1995). That is,

$$\tau = a_w (u_a - u_w) \tan \phi' \quad [9]$$

where: $a_w = A_{dw} / A_{tw}$

A_{dw} = area of water corresponding to any degree of saturation.

A_{tw} = total area of water at saturation

The normalized area of water, aw, decreases as the matric suction increases. The chain rule of differentiation on Eq. [9] shows that there is two components of shear strength change associated with a change in matric suction.

$$d\tau = \tan \phi' \left[a_w \, d(u_a - u_w) + (u_a - u_w) da_w\right] \quad [10]$$

The normalized area of water in the soil, a_w, may be assumed to be proportional to the normalized volumetric water content at a particular suction value by applying Green's theorem, (Fung, 1977) [i.e., $\Theta(u_a - u_w)$ which is equal to $\theta(u_a - u_w) / \theta_s$]. The normalized area of water can be defined by the following equation,

$$a_w = [\Theta(u_a - u_w)]^\kappa \quad [11]$$

where: $\Theta(u_a - u_w)$ = normalized volumetric water content as a function of matric suction, and

κ = a soil parameter dependent upon the soil type.

Then, substituting Eq. [11] into Eq. [10] gives

$$d\tau = \tan \phi' \left\{[\Theta(u_a - u_w)]^\kappa + \kappa(u_a - u_w)[\Theta(u_a - u_w)]^{\kappa - 1} d\Theta(u_a - u_w)\right\} d(u_a - u_w) \quad [12]$$

Integrating Eq. [11] yields,

$$\tau(u_a - u_w) = C + \tan \phi' \int_0^{u_a - u_w} \left\{[\Theta(u_a - u_w)]^\kappa + \kappa(u_a - u_w)[\Theta(u_a - u_w)]^{\kappa - 1} d\Theta(u_a - u_w)\right\} d(u_a - u_w) \quad [13]$$

where: C = constant of integration.

The constant of integration, *C*, in Eq. [13] is the shear strength of the soil at zero suction (i.e., the saturated shear strength). Therefore,

$$C = \tau(0) = c' + (\sigma_n - u_w)\tan\phi' \qquad [14]$$

where: $u_w = u_a$ (i.e., at saturation)
c' = effective cohesion,
ϕ' = effective angle of internal friction.

Substituting Eq. [14] into Eq. [13] gives the following shear strength expression as a function of matric suction and the effective angle of internal friction, ϕ',

$$\tau(u_a - u_w) = c' + (\sigma_n - u_a)\tan\phi' + \tan\phi' \int_0^{u_a-u_w} \left\{[\Theta(u_a - u_w)]^{\kappa} + \kappa(u_a - u_w)[\Theta(u_a - u_w)]^{\kappa-1} d\Theta(u_a - u_w)\right\} d(u_a - u_w) \qquad [15]$$

where: $\Theta(u_a - u_w) = \theta(u_a - u_w)/\theta_s$, and
$\theta(u_a - u_w)$ = the volumetric water content at any suction as given by Eq. [8].

The normalized volumetric water content, $\Theta(u_a - u_w)$, in Eq. [17] is defined by the soil-water characteristic function and can be obtained from Eq. [8]. In other words, Equation [15] can be used to predict the shear strength function of an unsaturated soil using the soil-water characteristic curve and the *saturated* shear strength parameters.

Equation [15] can be written in a different form as follows.

$$\tau = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)[\Theta(u_a - u_w)]^{\kappa}\tan\phi' \qquad [16]$$

This equation is found by substituting equation [11] into equation [9]. Equation [16] will produce the same shear strength curve as equation [15]. The simple form of equation [16] now allows for the easy substitution of a normalized soil-water characteristic curve. The other advantage of equation [16] is that the difficult integration shown in equation [15] is avoided. This allows equation [16] to work well with a number of soil-water characteristic equations.

Comparison of Theory to Example Data

Consider two different soils, Soil 1 and Soil 2, shown in Fig. 1. The soil-water characteristic curves are typical of a medium and fine-grained sand, respectively. Soil 1 has an effective cohesion of 0.0 kPa, an effective angle of internal friction of 32.0° and an air entry value of 20 kPa. Soil 2 has an effective cohesion of 0.0 kPa, an effective angle of internal friction of 25.0° and an air entry value of 60 kPa. The predicted shear strength curves for Soil 1 and Soil 2 using Eq. [16] are shown in Fig. 2. It can be seen that the shear strength of both soils increases linearly at the rate of ($\tan\phi'$) up to the air entry values of the soils. Beyond the air entry values, the rate of change of shear strength with matric suction decreases. The change in shear strength with respect to suction is in accordance with Eq. [12].

The shapes of the shear strength curves with respect to matric suction are similar to those measured by Donald (1956). Donald's test results for several sands are shown in Fig. 3. In each case the shear strength increases with suction and then drops off to a lower value. A similar behavior was observed in the testing of a fine to medium-grained decomposed tuff from Hong Kong (Fig. 4). The results indicate that at low confining pressures the shear strength may rise and then start to fall with increasing suction. At higher confining pressures the shear strength shows a continual rise in strength with increasing suction. These results also illustrate the importance of applying an appropriate confining pressure to the soil when measuring the soil-water characteristic curve.

$$\tan\phi^b = \frac{d\tau}{d(u_a - u_w)} = \left\{[\Theta(u_a - u_w)]^{\kappa} + \kappa(u_a - u_w)[\Theta(u_a - u_w)]^{\kappa-1} d\Theta(u_a - u_w)\right\}\tan\phi' \quad [17]$$

The ratio between the two friction angles (i.e., ϕ' and ϕ^b) can be shown as a function of the normalized water content.

$$\beta = \frac{\tan\phi^b}{\tan\phi'} = [\Theta(u_a - u_w)]^{\kappa} + \kappa(u_a - u_w)[\Theta(u_a - u_w)]^{\kappa-1} d\Theta(u_a - u_w) \quad [18]$$

Equations [17] and [18] show that the angle, ϕ^b, is equal to the effective angle of internal friction, ϕ', up to the air entry value of the soil (i.e., $\Theta(u_a - u_w) = 1$). Beyond the air entry value, ϕ^b decreases as the matric suction increases (Fig. 2). This is in agreement with experimental observations (Gan, et al., 1988).

Equation [16] has been tested using experimental data from a completely decomposed tuff from Hong Kong (Gan and Fredlund, 1992). A best-fit soil-water characteristic curve using Eq. [8] is found by using a curve fitting program to match the measured water contents at various matric suction values (Fig. 5). The shear strength function is calculated from Eq. [16]. The predicted shear strength values, along with the measured shear strength values are shown in Fig. 6. The parameters used in the model are listed in Table 1. The value of the soil parameter, κ, was set to 1 for the prediction. The model with κ equal to 1, appears to give satisfactory predictions for sandy soils. The value of κ generally increases with the plasticity of the soil and can be greater than 1.0.

The value of κ affects the rate at which the angle ϕ^b decreases as the matric suction exceeds the air entry value of the soil. The effect of κ on the shear strength function of a soil with a soil-water characteristic curve defined in Figures 7 and 8, is shown in Fig. 9. The values of κ range from 1.0 to 3.0. The influence of κ on the shape of the shear strength function occurs once the air entry value of the soil is exceeded. In the example shown, a value of κ equal to 2.0 shows that the shear strength envelope becomes essentially horizontal shortly after the air entry value is exceeded. The variable κ can be visualized as an indication of the relationship between the volumetric representation of water in the voids and the area representation of water in the voids as represented by an unbiased plane passed through the soil mass.

Close-Form Solutions

Closed-form solutions for shear strength functions are developed for two cases, using the empirical equations proposed by McKee and Bumb (1984) and Brooks and Corey (1964), respectively.

The following exponential relationship for the soil-water characteristic curve is suggested by McKee and Bumb (1984) for the case where the suction is greater than the air entry value (i.e., $(u_a-u_w) > (u_a-u_w)_b$). A sample plot of the equation proposed by McKee and Bumb (1984) can be seen in Fig. 10.

$$\Theta = e^{-\left[\frac{(u_a-u_w)-(u_a-u_w)_b}{f}\right]} \quad [19]$$

where: $(u_a-u_w)_b$ = the air entry value (also known as the bubbling pressure), f = a fitting parameter.

Equation. [19] describes the soil-water characteristic curve for suction values greater than the air entry value. The normalized volumetric water content, Θ, is assumed to be constant in the range from zero soil suction to the air entry value of the soil. For simplicity, the soil parameter, κ was assumed to be equal to 1. Substituting Eq. [19] into Eq. [16] gives the closed-form equation,

$$\tau = c' + (\sigma - u_a)\tan\phi' + \left[e^{-\left[\frac{(u_a - u_w) - (u_a - u_w)_b}{f}\right]}\right]^{\kappa}(u_a - u_w)\tan\phi' \qquad [20]$$

Sample plots of Eq. [20] showing the effect of varying the air entry value and f parameter are shown in Figures 11 and 12.

The soil-water characteristic curve given by Brooks and Corey (1964) can be expressed in the following form for the case where the suction is greater than the air entry value (i.e., $(u_a - u_w) > (u_a - u_w)_{AEV}$). A sample plot of Eq. [22] can be seen in Fig. 13.

$$\Theta = \left(\frac{(u_a - u_w)_{AEV}}{(u_a - u_w)}\right)^{f'} \qquad [22]$$

where: $(u_a - u_w)_{AEV}$ = the air entry value,
f' = a fitting parameter.

Equation [22] is valid for matric suctions greater than the air entry value (i.e., the value of Θ is assumed to be a constant up to the air entry value). Substituting Eq. [22] into Eq. [16] gives,

$$\tau = c' + (\sigma_n - u_a)\tan\phi' + \left(\frac{(u_a - u_w)_{AEV}}{(u_a - u_w)}\right)^{f'}(u_a - u_w)\tan\phi' \qquad [23]$$

Sample plots using Eq. [23] with varying f' and air entry values are shown in Figures 14 and 15. The parameters f and f' appear to have similar effects on the shear strength function (Fig. 12 and Fig. 15) as the parameter κ (Fig. 9). The parameters f and f' can therefore be expressed in terms of the parameter κ, thus eliminating one additional parameter from Eqs. [20] and [23].

Alternate Solution to the General Shear Strength Equation for an Unsaturated Soil

The experimental data used to illustrate the use of Eq. 15 was from a sandy soil (i.e., a decomposed tuff from Hong Kong). The data set showed a good fit with the parameter κ, set equal to 1. However, for highly plastic soils, the parameter κ, is greater than 1. At present, its magnitude is an unknown variable.

Attempts to best-fit other data sets have shown that it is possible to always leave the variable, κ, at 1.0 but change the upper limit of integration to reflect the soil suction near residual conditions. Unfortunately, the best-fit of the shear strength data, often occurs when the residual conditions vary from those used in the best-fit of the soil-water characteristic curve. In other words, there may not be a common residual suction value for both the soil-water characteristic data and the shear strength data. More data sets are required, along with further best-fit regression analyses, in order to better understand how best to predict the shear strength of an unsaturated soil.

Conclusions

A model is proposed for the prediction of the shear strength of an unsaturated soil. The model makes use of the soil-water characteristic curve and the saturated shear strength properties of the soil to predict the shear strength. The use of the model is illustrated using experimental data for a decomposed tuff soil from Hong Kong. The predicted shear strength curve shows good agreement with measured data; however, there is an additional soil parameter, κ; which becomes greater than 1 as the plasticity of the soil increases.

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REFERENCES

Brooks, R.H. and A.T. Corey, 1964. "Hydraulic Properties of Porous Medium", Colorado State Univ., Fort Collins. Hydrology Paper No. 3, 27 p.

Croney, D. and J.D. Coleman, 1961. "Pore Pressure and Suction in Soils", In Proceedings of the Conference on Pore Pressure and Suction in Soils, Butterworths, London, pp.31-37.

Donald, I.B., 1956. "Shear Strength Measurements in Unsaturated Non-Cohesive Soils With Negative Pore Pressures", Proceedings of 2nd Austraulia-New Zealand Conference of Soil Mechanics and Foundation Engineering, Christchurch, New Zealand, pp.200-205.

Fredlund, D.G., Vanapalli, S., Xing, A. and Pufahl, D.E., (1995), "Predicting the Shear Strength Function for Unsaturated Soils using the Soil-Water Characteristic Curve", Proc. of the First Int. Conf. on Unsaturated Soils, Paris, France, pp. 63-69

Fredlund, D.G., (1964) "Comparison of Soil Suction and One-Dimensional Consolidation Characteristics on a Highly Plastic Clay", Nat. Res. Council Tech. Report No. 245, Div. of Bldg. Res., Ottawa, Ontario, Canada, 26p., July

Fredlund, D.G. (1995) "The Stability of Slopes with Negative Pore-Water Pressures". The Ian Boyd Donald Symposium on Modern Developments in Geomechanics, edited by C.M. Haberfield, Monash University, Department of Civil Engineering, Clayton, Victoria 3168, Australia, pp. 99-116.

Fredlund, D. G., and N. R. Morgenstern, 1977. "Stress State Variables for Unsaturated Soils", ASCE J. Geotech. Eng. Div. Vol. 103, GT5, pp. 447-466.

Fredlund, D.G. and Anqing Xing, 1994. "Equations for the Soil-Water Characteristic Curve", Canadian Geotechnical Journal, Vol. 31, pp. 521-532.

Fredlund, D.G. and H. Rahardjo, 1993a. "Soil Mechanics for Unsaturated Soils", John Wiley and Sons, New York, 517 p.

Fredlund, D.G. and H. Rahardjo, 1993b. "An Overview of Unsaturated Soil Behaviour", Proceedings of ASCE Specialty Series on Unsaturated Soil Properties, Dallas, TX, October 24-28, pp. 1-31.

Fredlund, D. G., N. R. Morgenstern, and R. A. Widger, 1978. "The Shear Strength of Unsaturated Soils", Can. Geotech. J., Vol. 15, No. 3, pp. 313-321.

Fredlund, D.G., Anqing Xing and Shangyan Huang, 1994. "Predicting the Permeability Function for Unsaturated Soils Using the Soil-Water Characteristic Curve", Canadian Geotechnical Journal, Vol. 31, pp. 533-546.

Fung, Y.C. 1977, "A First Course in Continuum Mechanics", Prentice-Hall Inc., Englewood Cliffs, New Jersey.

Gan, J.K-M., D.G. Fredlund and H. Rahardjo, 1988. "Determination of the Shear Strength Parameters of an Unsaturated Soil Using the Direct Shear Test", Canadian Geotechnical Journal, Vol. 25, No. 8, pp. 500-510.

Gan, J.K-M., and D.G. Fredlund, 1992. "Direct Shear Testing of a Hong Kong Soil under Various Applied Matric Suctions", GEO Report No. 11, Geotechnical Engineering Office, Civil Engineering Department, Hong Kong, 587 pp.

Gan, J,K-M., and D.G. Fredlund, 1996. "Shear Strength Characteristics of Two Saprolitic Soils", Submitted to Canadian Geotechnical Journal.

Karube, D., 1988. "New Concept of Effective Stress in Unsaturated Soil and Its Proving Test", Advanced Triaxial Testing of Soil and Rock, ASTM STP 977, American Society for Testing and Materials, Philadelphia, pp. 539-552.

Lamborn, M.J., 1986. "A Micromechanical Approach to Modeling Partly Saturated Soils", M.Sc. Thesis, Texas A & M University, Texas.

McKee, C.R. and A.C. Bumb, 1984. "The Importance of Unsaturated Flow parameters in Designing a Monitoring System for a Hazardous Wastes and Environmental Emergencies" Hazardous Materials Control Research Institute National Conference, Houston, TX, March 1984 pp. 50-58.

Papagiannakis, A. T. and D. G. Fredlund, 1984. "A Steady State Model for Flow in Saturated-Unsaturated Soils", Can. Geotech. J., Vol. 21, No. 13, pp. 419-430.

Peterson, R.F.W., 1988. "Interpretation of Triaxial Compression Test Results on Partially Saturated Soils", Advanced Triaxial Testing of Soil and Rock, ASTM STP 977, American Society for Testing and Materials, Philadelphia, pp. 512-538.

Richards, B.G., 1965. "Measurement of the Free Energy of Soil Moisture by the Psychrometric Technique Using Thermistors, in Moisture Equilibria and Moisture Changes in Soils beneath Covered Areas", Butterworth, Australia, pp. 39-46.

Russam, K. and C.D. Coleman, 1961, "The Effect of Climatic Factors on Subgrade Moisture Conditions", Geotechnique, vol. XI, No. 1, pp. 22-28.

Satija, B. S., 1978. "Shear Behaviour of Partly Saturated Soils", Ph. D. Thesis, Indian Inst. of Tech., Delhi, India.

Terzaghi, K., 1936. "The Shear Resistance of Saturated Soils", Proc. 1st Int. Conf. Soil Mech. Found. Eng., Cambridge, U. S. A., Vol. 1, pp.54-56

Toll, D.G., 1990. "A Framework for Unsaturated Soil Behaviour", Geotechnique, Vol. 40, No. 1, pp. 31-44.

Williams, P. J., 1982. "The Surface of the Earth, An Introduction to Geotechnical Science", Longman Inc., New York.

Table 1. Soil Properties and Fitting Parameters for the Hong Kong Soil US-1.

Equation [16]			Equation [7]			
$\tan\phi'$	c' (kPa)	κ	a (kPa)	n	m	h_r (kPa)
.8012	0.0	1	110.48	2.015	10.618	3000

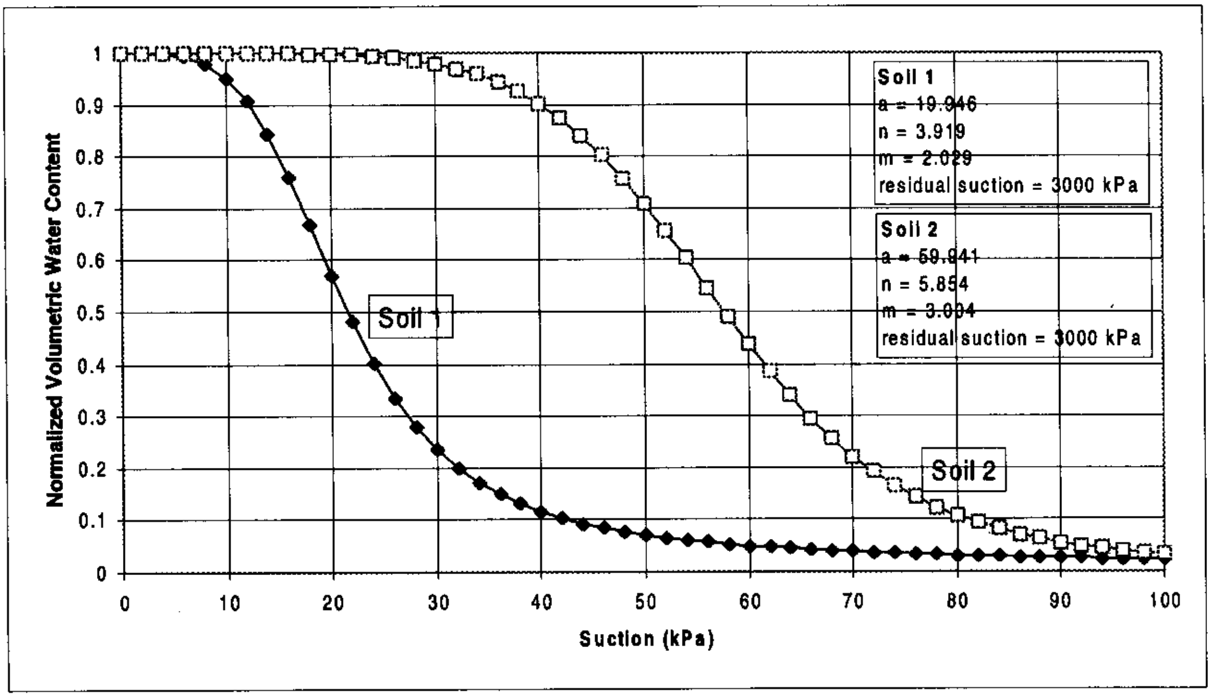

Figure 1 Two sample soil-water characteristic curves from Equation [7].

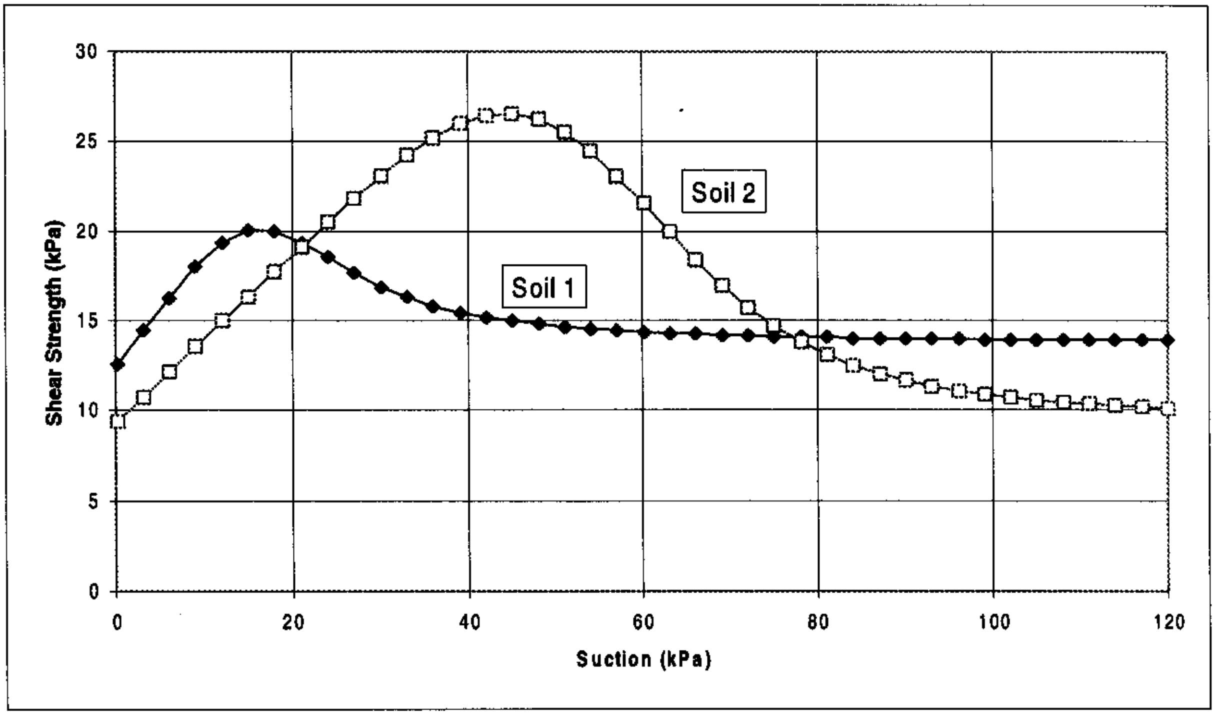

Figure 2 Predicted shear strength curves using equation [16] and the soil-water characteristic curves in Figure 1

Figure 3 Results of direct shear tests on sands under low matric suctions (modified from Donald, 1956)

Figure 4 Peak shear stress versus matric suction envelope for the completely decomposed fine ash tuff (Gan & Fredlund, 1996)

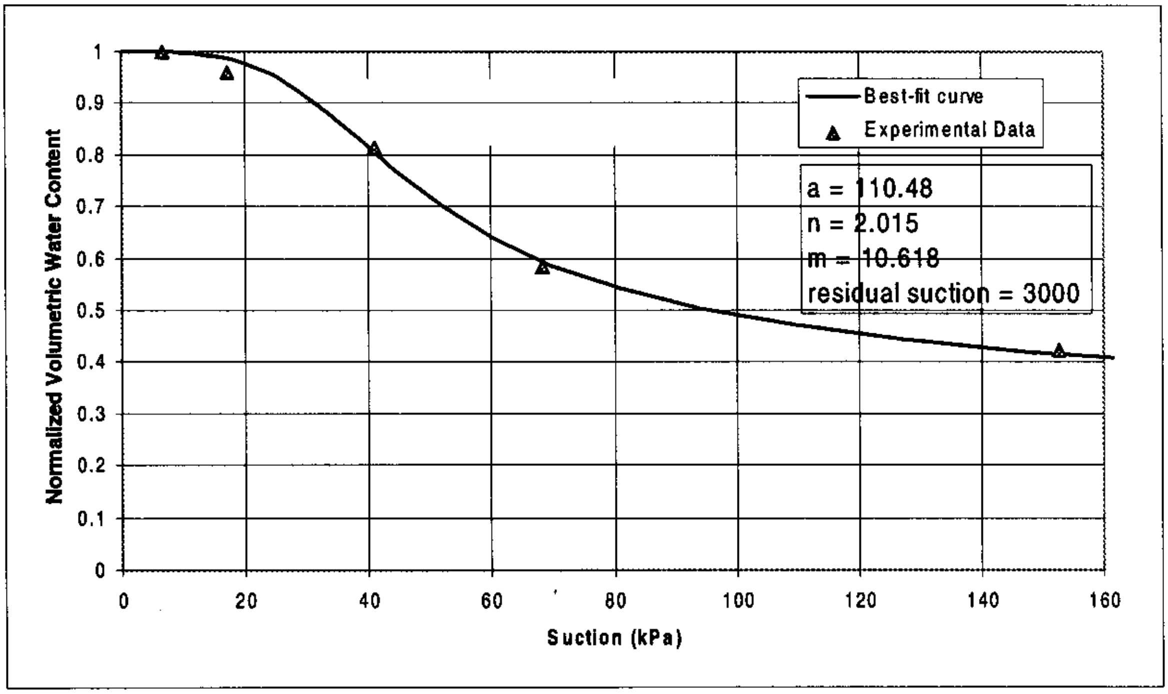

Figure 5 Soil-water characteristic curve for a completely decomposed tuff (specimen US-1) from Hong Kong, from experimental values (from Gan and Fredlund, 1992) and from calculations using equation [7]

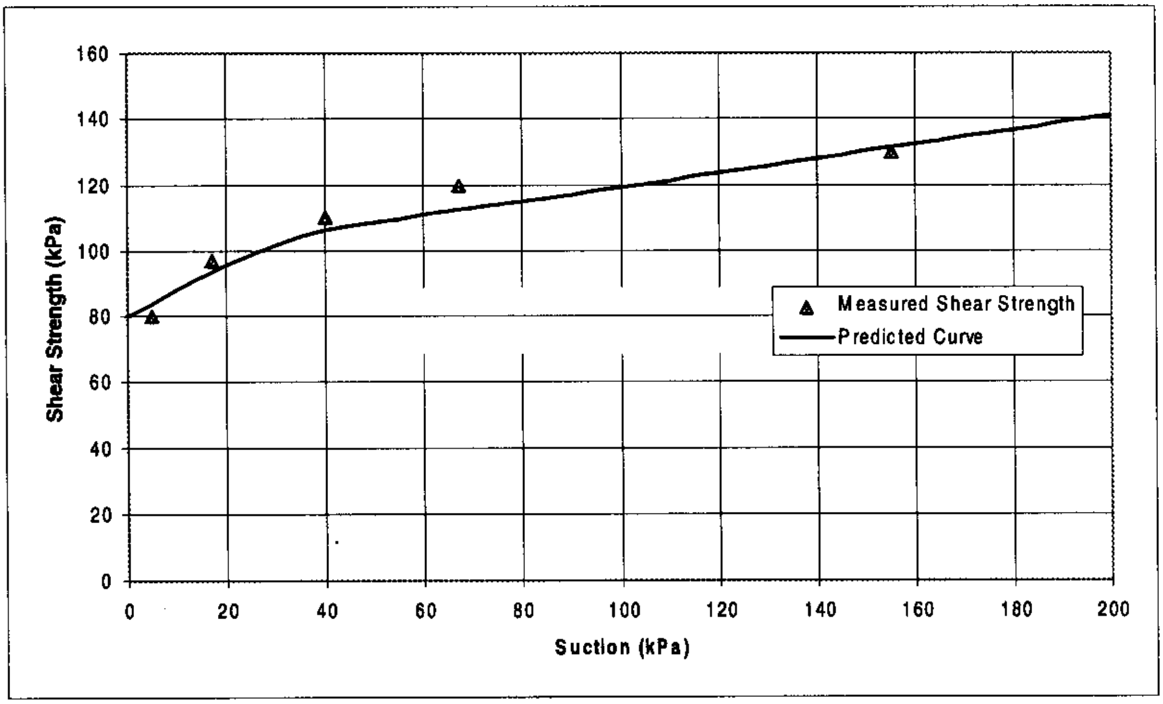

Figure 6 Comparison of the predicted shear strength curve with the experimental shear strength data for the completely decomposed tuff (specimen US-1) from Hong Kong (from Gan and Fredlund, 1992)

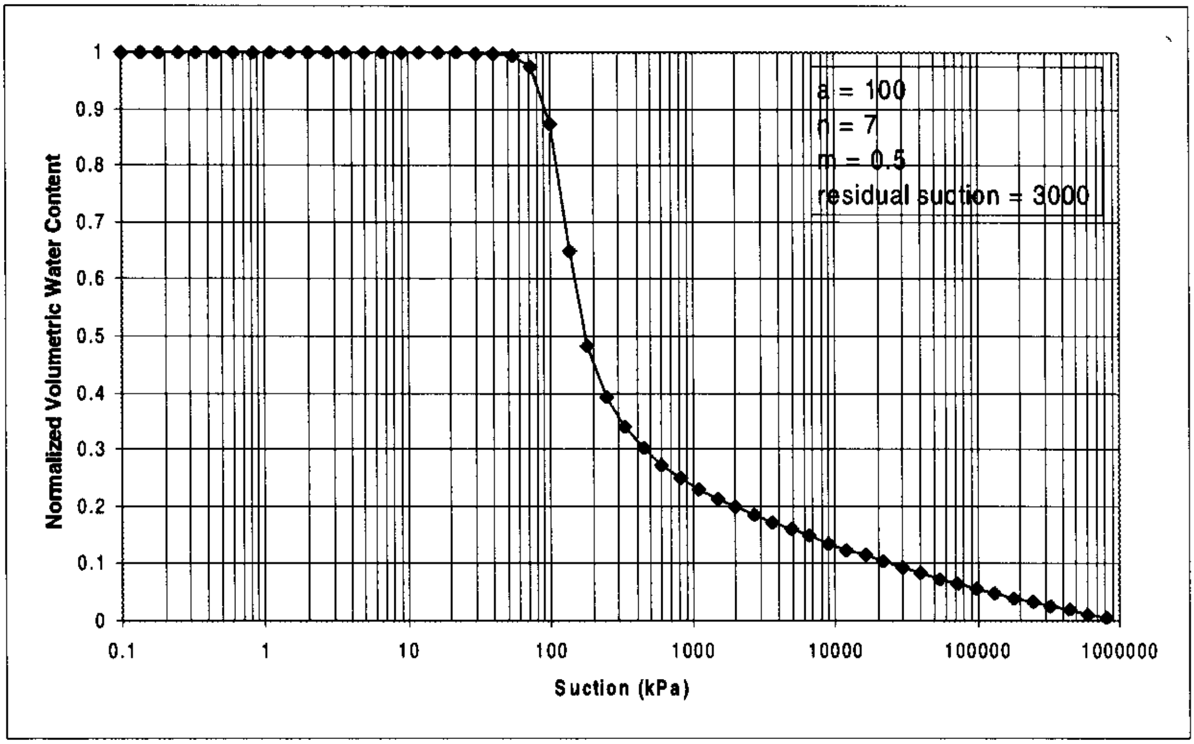

Figure 7 The normalized soil-water characteristic curve over the entire range of suction values.

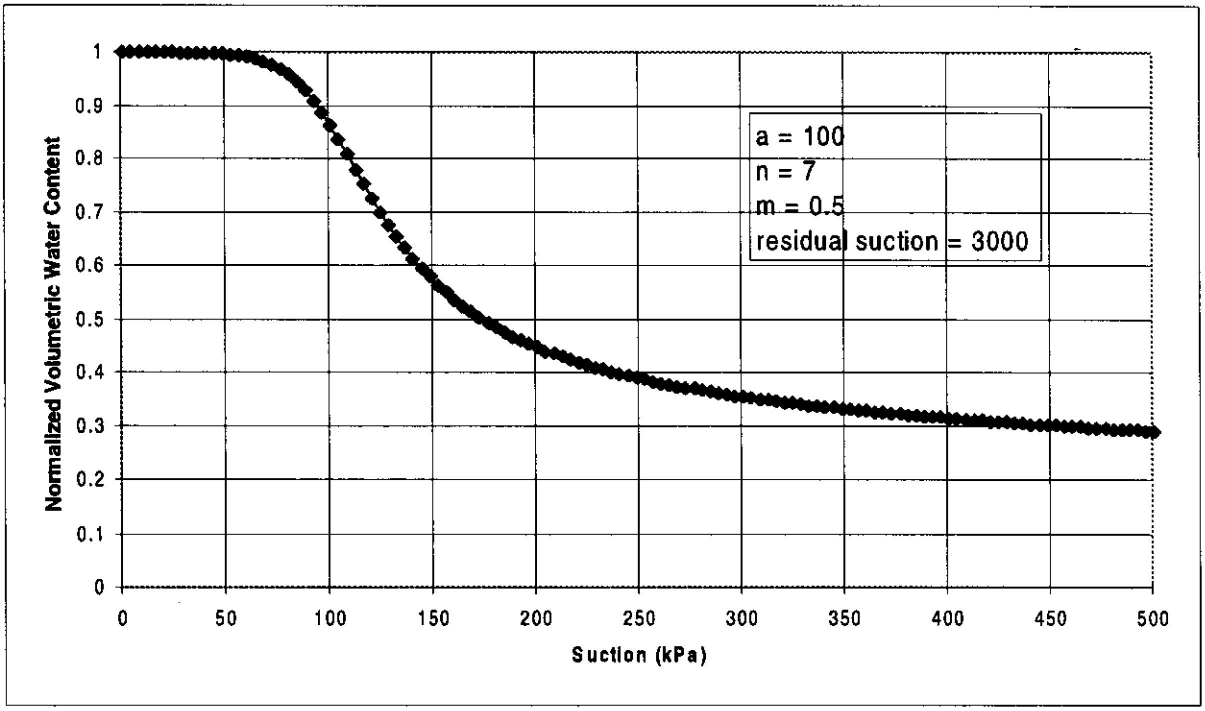

Figure 8 Effect of the parameter, kappa, on the shear strength function of a soil; the soil-water characteristic curve of the soil

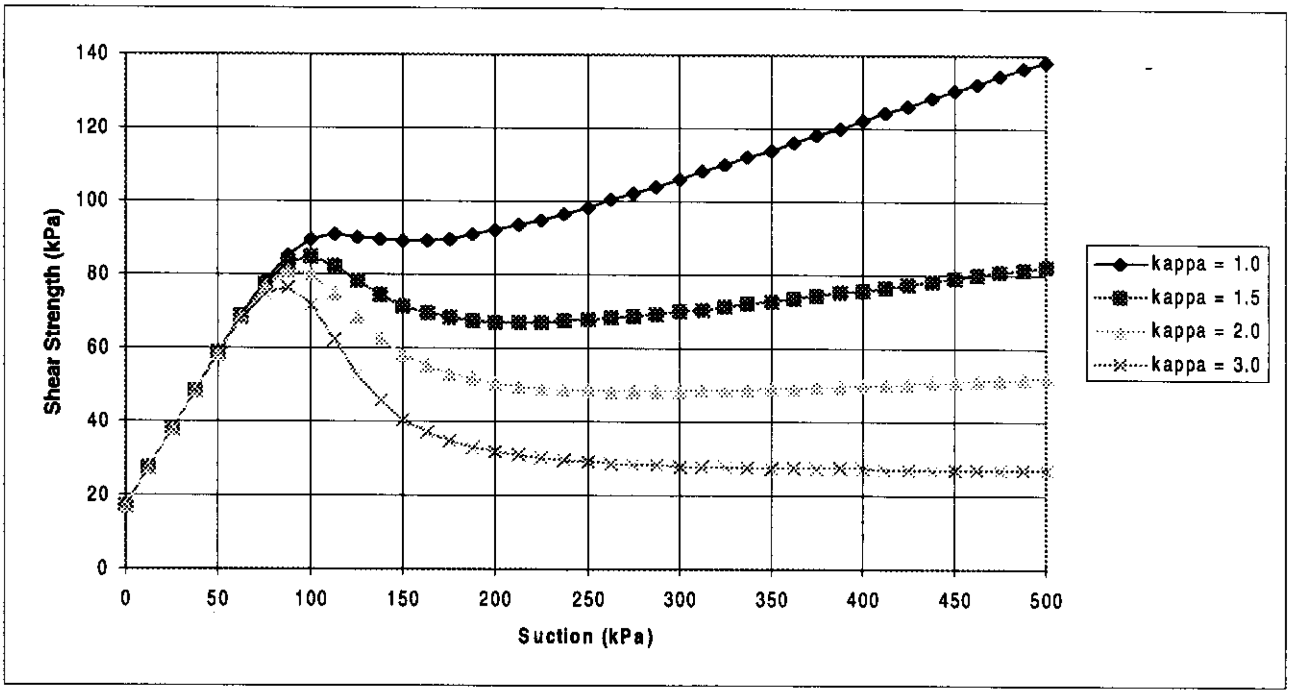

Figure 9 Effect of the parameter, kappa, on the shear strength function of a soil; shear strength functions showing the effect of varying kappa

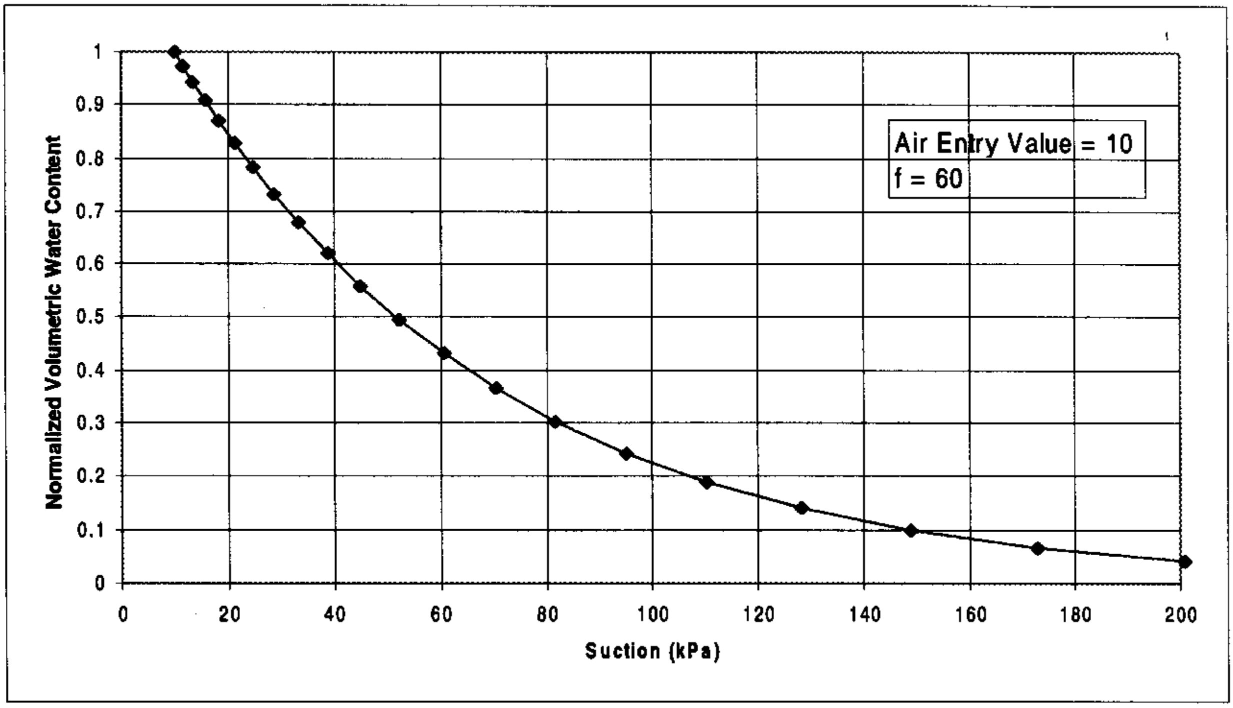

Figure 10 A sample plot of McKee and Bumb's equation (1984)

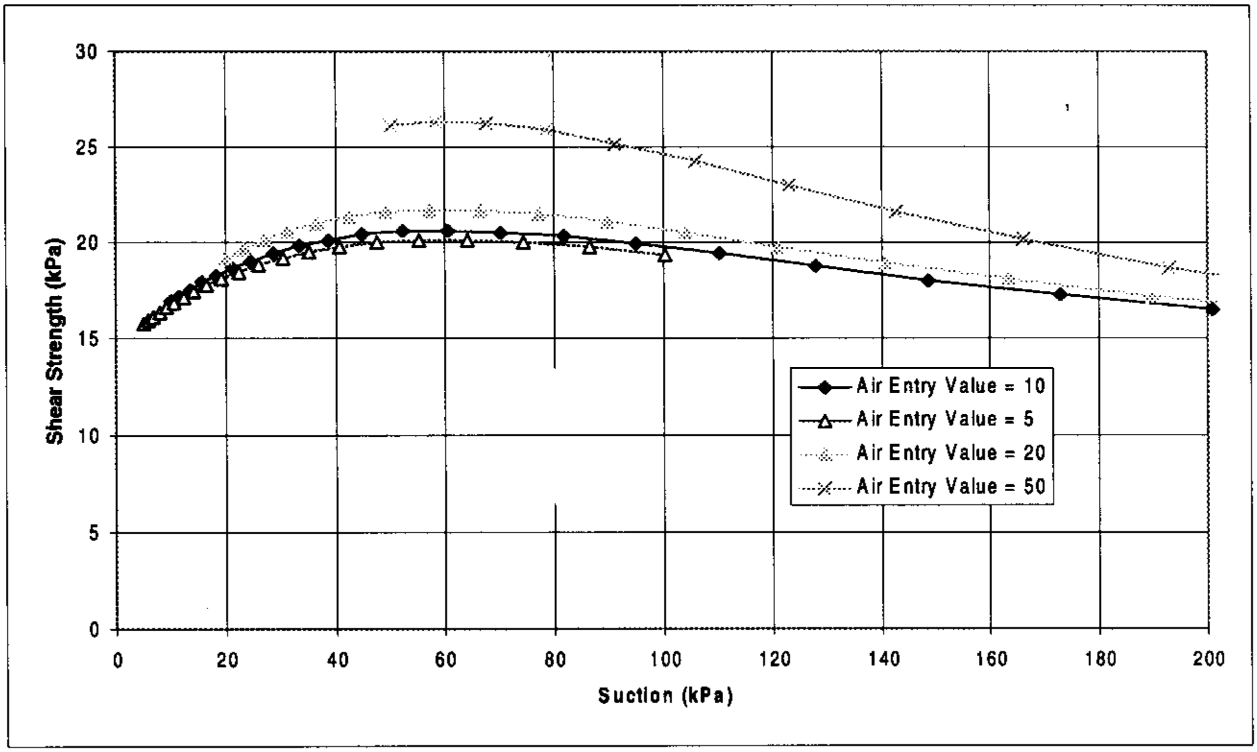

Figure 11 Shear strength function predicted using the McKee and Bumb equation (1984); Illustrating the effect of varying the air-entry value.

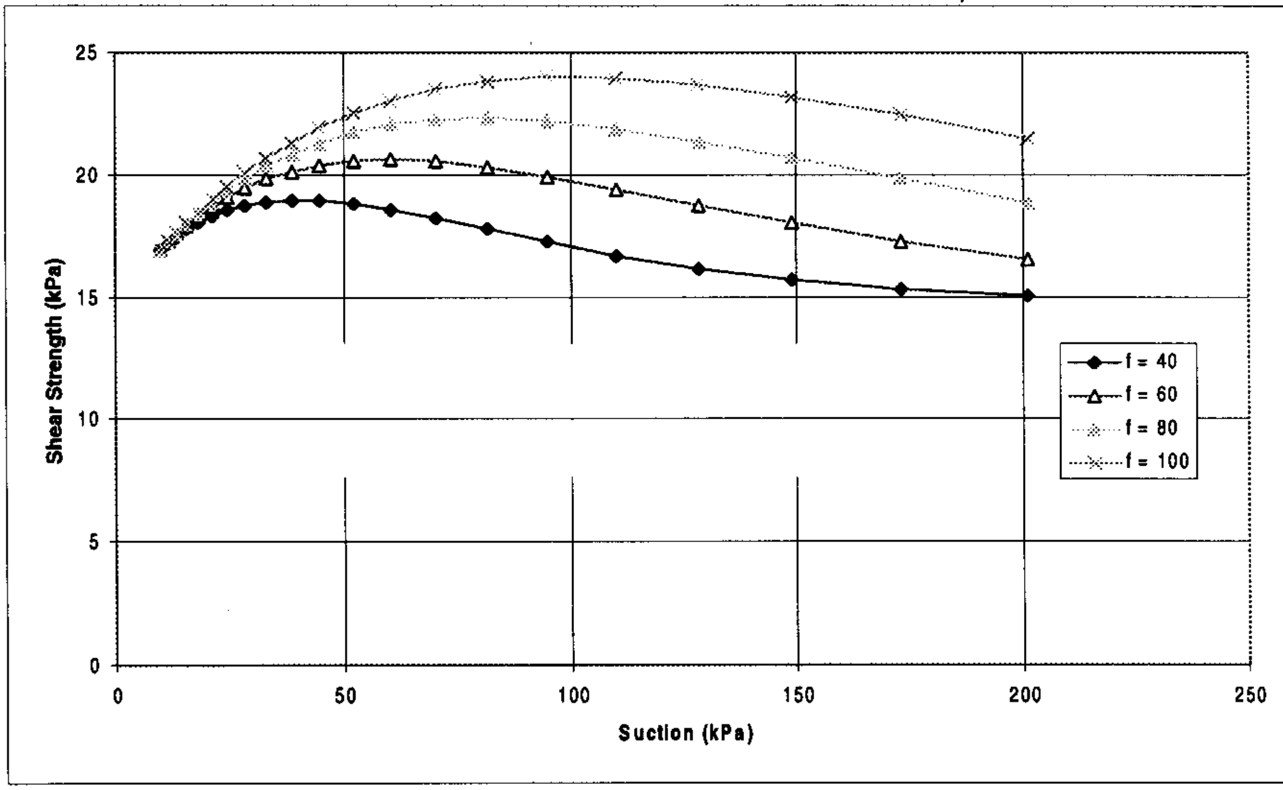

Figure 12 Shear strength function predicted using the McKee and Bumb equation (1984); illustrating the effects of varying the f parameter.

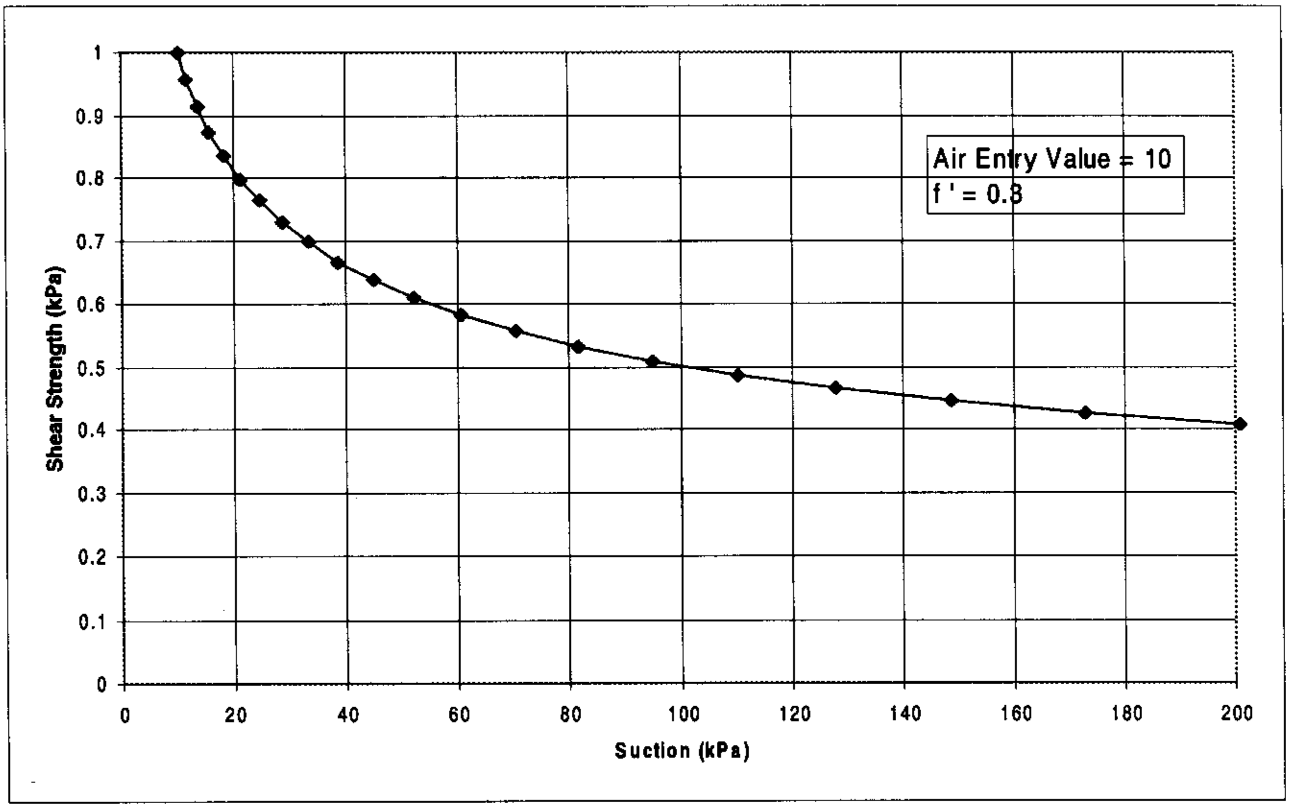

Figure 13 A sample soil-water characteristic curve using the Brooks and Corey equation (1964)

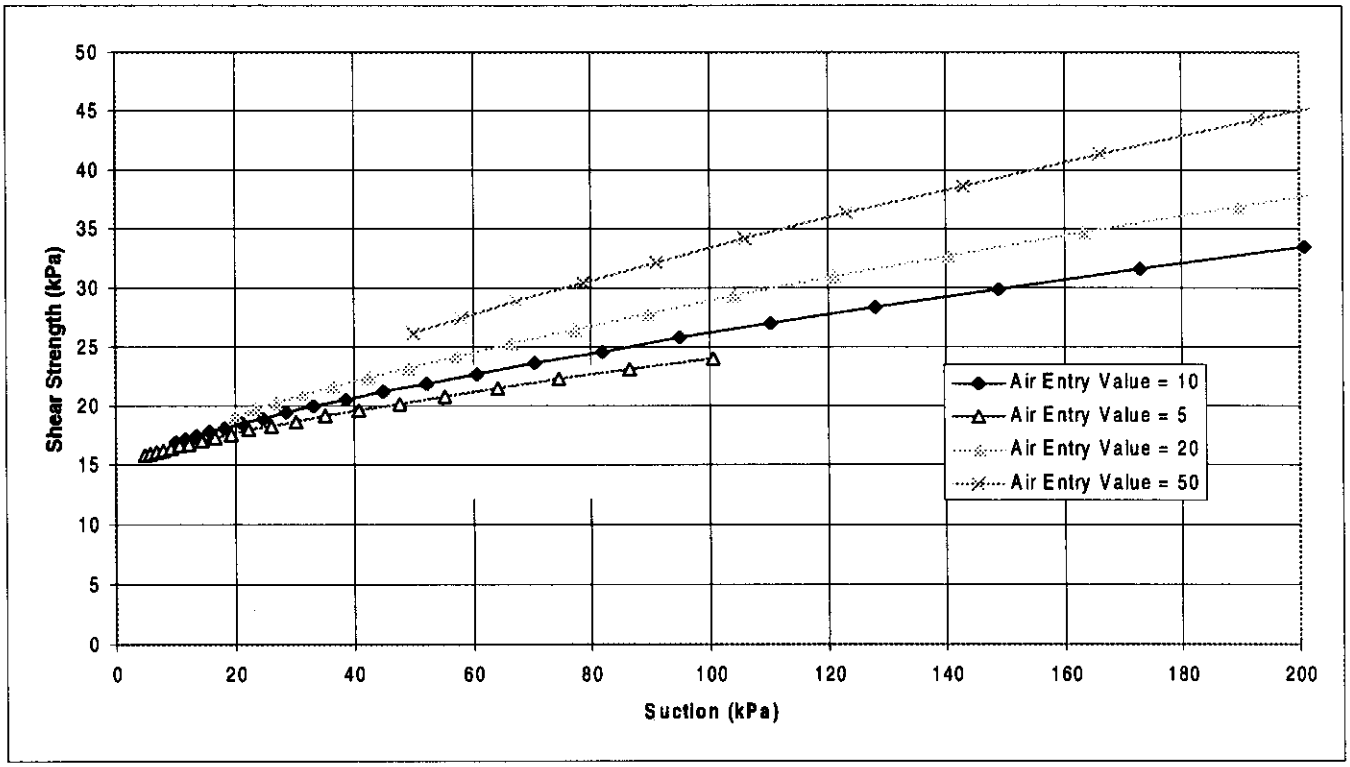

Figure 14 Shear strength equation using the Brooks and Corey equation (1964) for the soil-water characteristic curve and illustrating the effects of varying the air entry value.

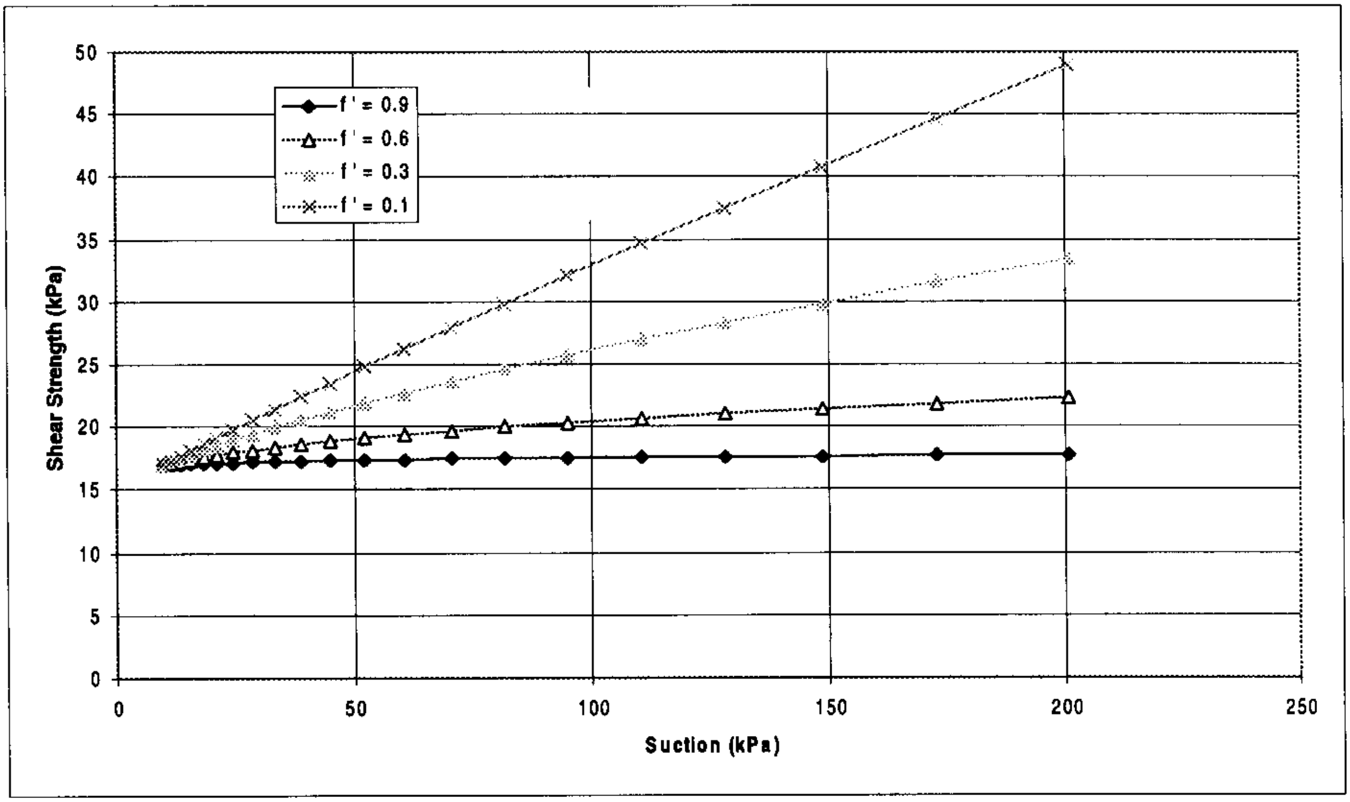

Figure 15 Shear strength equation developed using the Brooks and Corey equation (1964) for the soil-water characteristic equation and illustrating the effects of varying the f' parameter.

LIST OF FIGURES