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SEEPAGE MODELING IN A SATURATED/UNSATURATED SOIL SYSTEM

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ABSTRACT: This paper presents the application of general-purpose partial differential equation solvers associated soil property functions to the analysis of saturated-unsaturated seepage problem. Examples of two-dimensional steady state seepage, two-dimensional transient seepage through earth dam and three-dimensional seepage in a soil system are also presented.

Key Words: *seepage modeling, unsaturated soils, database, finite element method*

1. INRODUCTION

Seepage analysis forms an important and basic part of geotechnical engineering. Seepage analysis may be required in volume change prediction, ground water contamination control, slope stability analysis, and the design of earth structures such as dykes or dams.

The solution of the linear partial differential equation of flow was first proposed by Casagrande (1937) through the use of the graphical flownet method. This method is based on the assumptions that the soil is homogeneous and isotropic, and that water flows only in the saturated zone. The boundaries of the flow region must be defined in terms of head or no-flow. The flownet solutions proposed by Casagrande (1937) were for simple unconfined flow cases without flux boundary conditions.

In the late 1960's, the development and application of the digital computer to solving complex seepage problems came into prominence. Freeze (1971) proposed a finite difference model of flow through both the saturated and unsaturated soil regions. In the late 1970's, the increased computer power, combined with the finite element method became a powerful tool in solving steady state and transient saturated-unsaturated seepage problems. Several computer programs have been written for solving saturated-unsaturated modeling in engineering practice; examples being, MODFLOW[1] and SEEP/W[2].

Seepage analysis for unsaturated soils is mathematically characterized by a partial differential equation that is non-linear and soil properties that can be highly non-linear. As a result, the modeling of saturated-unsaturated soil systems has proven to be a challenge. The primary challenge in solving problems involving unsaturated soils is to develop a numerical software package that ensures convergence when solving seepage problems involving saturated-unsaturated soil systems (Fredlund, 1996).

In the last two decades, the development and application of the computer to solving complex problems has been extensive. An unsaturated soil problem involves the soil properties that are highly nonlinear, such as coefficient of permeability and water storage functions. The partial differential equations to be solved become highly nonlinear and require the input from persons specially trained in the area of mathematics. This has given rise to the use of general partial differential equation solvers that are designed to solve equations from many areas of engineering.

The solution of transient, unsaturated soil systems requires two non-linear soil property functions; the coefficient of permeability function, k_w, (also referred to as hydraulic conductivity function) and the water storage function, m_2^w (Fredlund and Rahardjo, 1993). Both soil property relationships can be written in terms of the negative pore-water pressure, u_w, (or soil suction, ψ).

Several general-purpose partial differential equation solvers have been developed for solving a variety of boundary value and/or initial value problems. Examples of these programs are PDEase2D[3], FlexPDE[4] and FEMLAB[5]. These programs, developed by mathematicians, have many features that are of interest to geotechnical engineers.

Finite element programs such as PDEase2D and FlexPDE have been used to analyze unsaturated soil problems of seepage (Fredlund, 1997; Thieu 1999), volume change (Hung, 2000), heat transfer (Pentland, 2000) and slope stability (Pham, 2001). While PDEase2D can analyze only two-dimensional problem, FlexPDE is extended to solve problems in three-dimensions.

Recently, a software package named SVFlux[6] has been introduced to allow seepage analysis in two dimensions and three dimensions. SVFlux has shown to be a useful tool for geotechnical engineers. SVFlux can also be coupled with a database package, SoilVision[7] to perform analysis without an extensive laboratory program. Soil properties can be obtained from a laboratory database of a variety of soils and geometry of three-dimensional problems can be inputted as surfaces and layers using survey data.

This paper presents the solutions of some seepage example problems to show the capability of the general partial differential equation solvers and the SVFlux software in solving two-dimensional and three-dimensional seepage in saturated/unsaturated soil systems.

2. THEORY OF SATURATED-UNSATURATED SEEPAGE

Water phase and air phase of an unsaturated soil are considered as fluids, therefore problems associated with unsaturated flow are two-phase flow problem. Two partial differential equations are required to rigorously describe the flow of air and water in an unsaturated soils (Fredlund, 1981). However, only flow of water is of interest in most engineering problems encountered in practice since air phase can be assumed to be continuous and atmospheric. The air phase can be occluded when degree of saturation is relatively high, but the air pressure will essentially be atmospheric and the effect of air flow is negligible. Freeze and Cherry (1979) stated that unsaturated seepage analysis involving only the water phase provides results that are accurate enough for almost all practical purpose.

Water flow is caused by a hydraulic head gradient. The hydraulic head (or total head) consists of the velocity head, pressure head and elevation head. The velocity head in a soil is negligible in comparison with the pressure head and elevation head, and the expression for total head can be written as follows:

$$h = \frac{u_w}{\gamma_w} + Y \qquad (1)$$

where: h = total head, u_w = pore-water pressure, γ_w = unit weight of water, and Y = elevation head above an arbitrary datum.

Darcy's law can be used to describe water flow through soils in both saturated and unsaturated condition (Richards, 1931). Darcy's law is stated as follows:

$$q = k_w i \qquad (2)$$

where: q = discharge per unit area, i = total head gradient, and k_w = coefficient of permeability.

The governing partial differential equation for seepage through a heterogeneous, anisotropic, saturated-unsaturated soil can be derived by satisfying conservation of mass for a representative elemental volume. If it is assumed that the total stress remains constant during a transient process, the differential equation can be written as follows for the three-dimensional transient case:

$$\frac{\partial}{\partial x}\left(k_{wx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_{wy}\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_{wz}\frac{\partial h}{\partial z}\right) = m_2^w \gamma_w \frac{\partial h}{\partial t} \qquad (3)$$

where k_{wx}, k_{wy}, and k_{wz} = coefficient of permeability of the soil in the x-, y-, and z-direction, respectively; and $m^w{}_2$ = the slope of the soil-water characteristic curve (i.e., water storage).

Equation 3 indicates that two soil properties (i.e., coefficient of permeability and water storage) are required to solve transient seepage problem associated with a saturated-unsaturated soil system. For steady state seepage, only the coefficient of permeability is required because the time dependent term in Eq. 3 disappears and the water storage drops out.

The water coefficient of permeability is a measure of the ability of soils to conduct water. The coefficient of permeability is a function of the volumetric water content, which is, in turn, a function of soil suction. The water storage is an indication of the amount of water taken or released by the soil as a result of a change in the pore-water pressure (i.e., soil suction) and it is the slope of the soil-water characteristic curve.

3. MAJOR FEATURES OF GENERAL PARTIAL DIFFERENTIAL EQUATION SOLVERS AND SVFLUX SOFTWARE

Major features of these newly developed solvers include:

- AutoCAD™[8] style CAD input,
- automatic mesh generation and refinement
- adaptive time step design and refinement
- ensuring convergence when solving nonlinear equations.
- allowing material properties to be input in a variety of forms. These forms can be categorized as a mathematical equation or a series of data points (Fig. 1). The series of data points could be handled in a number of different ways of input as shown in Fig. 1 and illustrated in Fig. 2.
- input 3D problems as surfaces and layers using survey data

In addition, when SVFlux coupled with Soilvision database program, soil properties can be selected or estimated from a laboratory database of over 6000 soils.

4. EXAMPLE PROBLEMS

Three example problems, the first associated with steady state seepage in two-dimensions, the second associated with transient state seepage in two dimensions, and the third associated with transient state seepage in three-dimensions, will be presented.

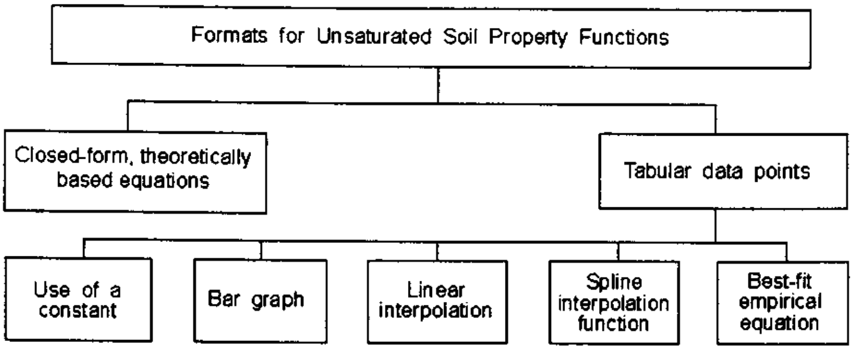

Figure 1. Summary of possible format for inputting unsaturated soil property functions

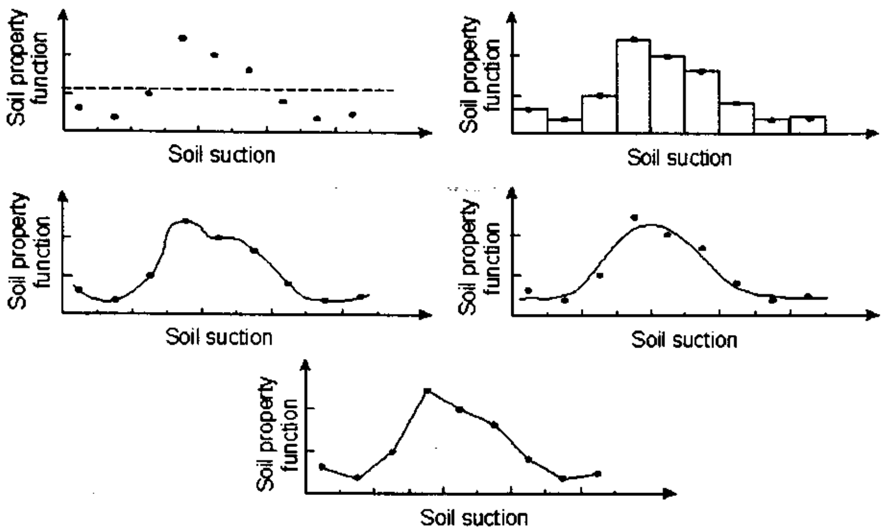

Figure 2. Illustration of formats for inputting unsaturated soil properties as a function of soil suction

Table 1. Permeability functions and fitted parameters for steady-state seepage analyses

	Permeability functions	Fitted parameters
Gardner (1958)	$k_w = \frac{k_s}{1+a\psi^n}$	$a = 1.969 \times 10^{-10}$, $n = 6.912$
van Genuchten-Burdine (1980)	$k_w = \frac{k_s}{\left[1+(a\psi)^n\right]^{\left(1-\frac{2}{n}\right)}}$	$a = 4.127 \times 10^{-2}$, $n = 9.401$ $m = 0.787$
van Genuchten-Mualem (1980)	$k_w = \frac{k_s}{\left[1+(a\psi)^n\right]^{\left(1-\frac{1}{n}\right)}}$	$a = 4.049 \times 10^{-2}$, $n = 8.852$ $m = 0.887$
Fredlund and Xing (1994)	$k_w = k_s \frac{\int_{ln(\psi)}^{b} \frac{\theta(e^y)-\theta(\psi)}{e^y}\theta'(e^y)dy}{\int_{ln(\psi_{aev})}^{b} \frac{\theta(e^y)-\theta_s}{e^y}\theta'(e^y)dy}$	$b = ln(1{,}000{,}000)$, y = dummy variable of integration representing the logarithm of suction, ψ_{aev} = soil suction at air entry value

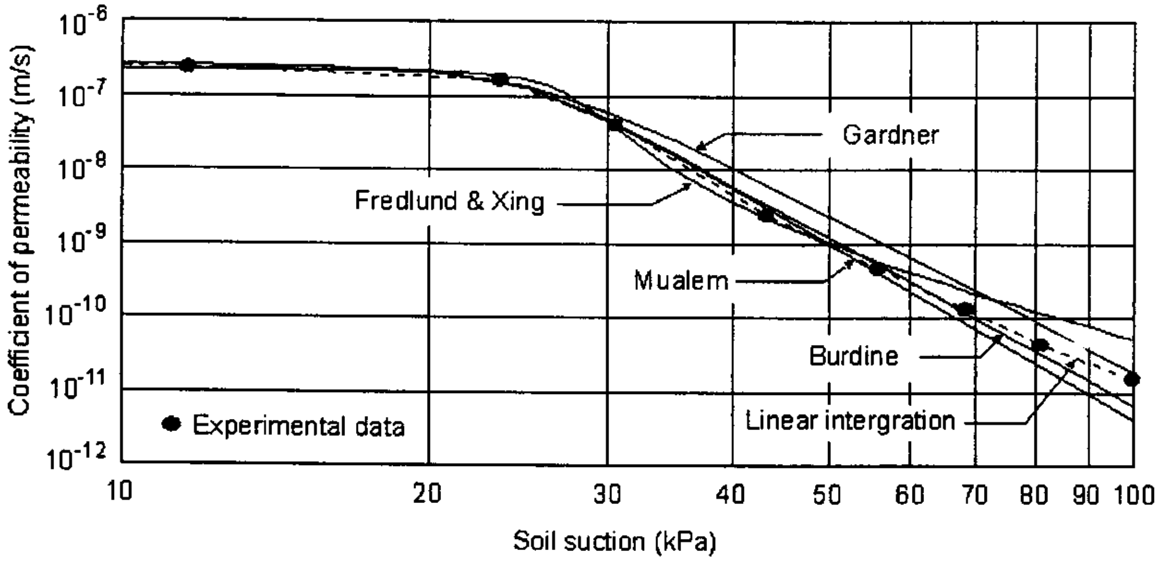

Figure 3. Specified permeability functions for the steady state seepage example

The soil material for the first two examples is assumed to be silt and isotropic with respect to the coefficient of permeability. Experimental data showing the coefficient of permeability versus matric suction, and the volumetric water content versus matric suction are obtained from Ho (1979). The saturated coefficient of permeability is 2.5×10^{-7} m/sec and the saturated volumetric water content is 0.381.

(1) Steady state seepage example

This example is presented to illustrate the different forms of input the coefficient of permeability when analyzing steady-state seepage through an isotropic earth dam with a horizontal drain. The permeability functions used are the Gardner (1958) equation, van Genuchten-Mualem (1980) equation, van Genuchten-Burdine (1980) equation, Fredlund and Xing (1994) equation and a series of data points. The parametric study was done to find the best-fit values for the permeability functions using MathCad[9] program. The permeability functions and their values of fitted parameters are shown in Table 1. Specified permeability functions together with the experimental data used to analyze the problem are presented graphically in Fig. 3.

The geometry, boundary conditions and finite element mesh used in running PDEase2D for steady-state seepage example are shown in Fig. 4. A maximum error of 0.1% was specified. The pore-water pressure distribution and flow vectors under

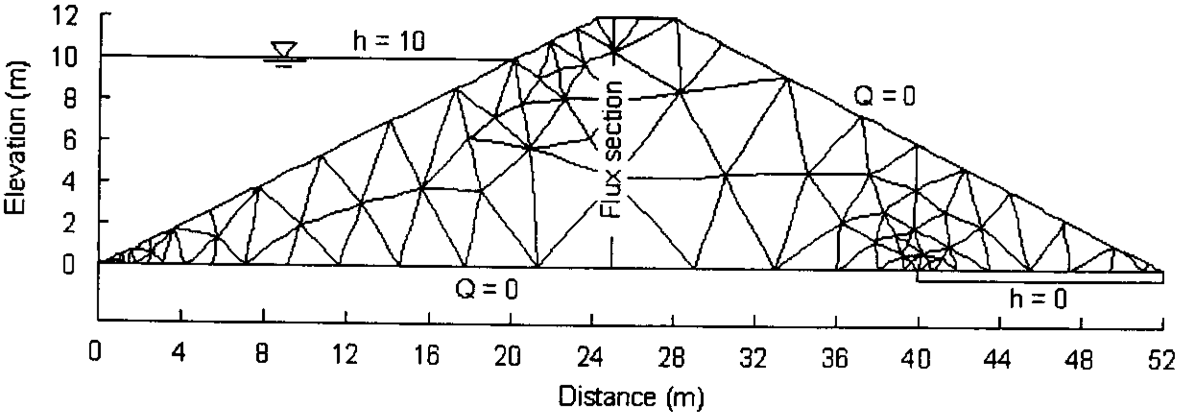

Figure 4. Geometry, boundary conditions and the finite element mesh for the steady state seepage example

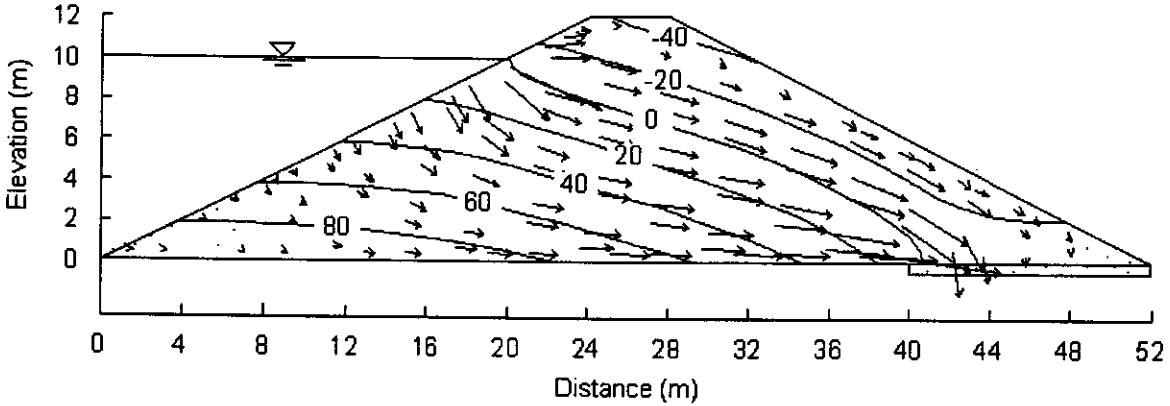

Figure 5. Pore-water pressure distribution and flow vectors for steady state seepage example

steady state seepage are shown in Fig. 5 for different forms of data input. The position of the total head contours and phreatic lines are the same for all permeability functions.

The coefficient of permeability function does not need to be specified precisely when computing the distribution of pore-water pressure. Therefore, an approximate permeability function is adequate for analysis purposes. However, the quantity of flux through a section is slightly different, as shown in Table 2. The differences shown in Table 2 are not significant from an engineering standpoint and the results would indicate that any one of several possible permeability functions would yield satisfactory results.

Table 2. Value of flux quantities using various permeability functions

Functions	Quantity of flux ($x 10^{-7} m^3/s$)	Deviation (%)
Gardner	7.449	0.8
van Genuchten-Burdine	7.432	0.5
van Genuchten-Mualem	7.430	0.5
Fredlund & Xing	7.376	0.2
Linear interpolation	7.272	1.6
Average	7.392	0

(2) Transient state seepage example

The second example is presented to show the different forms that can be used to input the coefficient of water storage when analyzing transient seepage through an isotropic earth dam with a horizontal drain. The base of the dam is selected as the datum. Initially, the dam is at steady-state conditions with the reservoir water level of 4 m above the datum. At a time assumed to be equal to zero, the water level in the reservoir is instantaneously raised to a level of 10 m above the datum.

Gardner (1958) equation is used to describe the permeability function. The water storage function is obtained by differentiating the soil-water characteristic curve. The soil-water characteristic curve is de scribed using the van Genuchten (1980) equation, the extended error function, the Fredlund and Xing (1994) equation and the step-wise values. The parametric study was done to find out the best-fit parameters for the water storage function using MathCad program. The software, SoilVision (1998) can also be used for this purpose. The derivatives of the soil-water characteristic curve with respect to matric suction and their best-fit values are shown in Table 3 and Fig. 6.

Table 3. Water storage functions and fitted parameters for transient seepage analyses

	Water storage functions	Fitted parameters
van Genuchten (1980)	$\frac{d\theta}{d\psi} = -\frac{\theta_s}{(1+(a\psi)^n)^m}(a\psi)^n \frac{mn}{\psi(1+(a\psi)^n)}$	$a = 4.391 \times 10^{-2}$, $n = 46.754$, $m = 0.036$
Extended error function	$\frac{d\theta}{d\psi} = Ce^{a(\lvert\psi-\omega\rvert)^n}$	$a = -0.183$, $n = 1.073$, $c = 0.028$, $\omega = 28.783$
Fredlund & Xing (1994)	$\frac{d\theta}{d\psi} = -\theta_s \left[\frac{1}{\ln\left[e+\left(\frac{\psi}{a}\right)^n\right]}\right]^m \frac{m}{\ln\left[e+\left(\frac{\psi}{a}\right)^n\right]}\left(\frac{\psi}{a}\right)^n \frac{n}{\psi\left[e+\left(\frac{\psi}{a}\right)^n\right]}$	$a = 26.127$, $n = 14.030$, $m = 0.622$

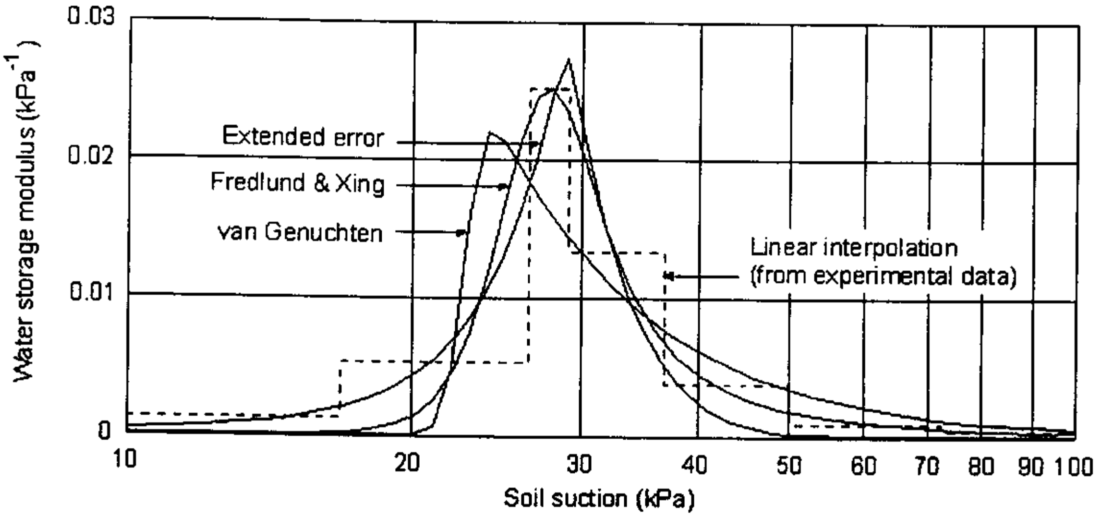

Figure 6. Water storage functions for transient seepage analyses

The geometry and boundary conditions for this example are shown in Fig. 7. A steady-state condition with water level of 4 m is first performed in order to get the distribution of initial pore-water pressure head. A maximum error limit of 0.5% was specified. The number of elements and nodes used in running this process varies with time from 181 to 1006, and from 428 to 2215, respectively.

The pore-water pressure distributions and the phreatic lines for elapsed time equal to 15 hours corresponding to different forms of inputting the data are shown in Fig. 8. The results indicate that the water storage functions defined are quite sensitive to the transient state seepage predictions at early times. The phreatic line obtained using various water storage functions would be closer at latter time steps, and approaches the same location at steady state conditions. The solution of the problem at steady state condition was presented previously in Fig. 5.

(3) Three-dimensional steady state seepage example

The third example presents the steady-state seepage modeling of a simulated tailings pit. The geometry is highly irregular and soil properties range from a fine silt material in the center of the basin to a coarse gravel on the outer edges of the problem. This problem is similar to the actual deposition of typical tailings pits at various mine sites in northern Canada. Tailings material is spigotted out over dykes at the edge of the basin. Coarser material is therefore deposited at the outer edges of the basin while the fines travel to the center of the basin before falling out of solution.

This deposition process creates a problem with complicated geometry and highly irregular soil properties which was previously difficult to solve. SVFlux simplifies the solution of such a problem by

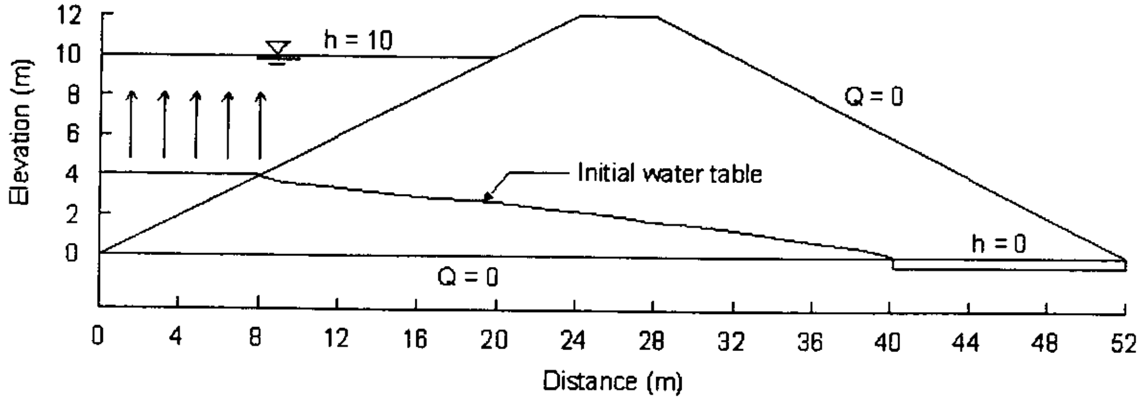

Figure 7. Geometry and boundary conditions for transient seepage example

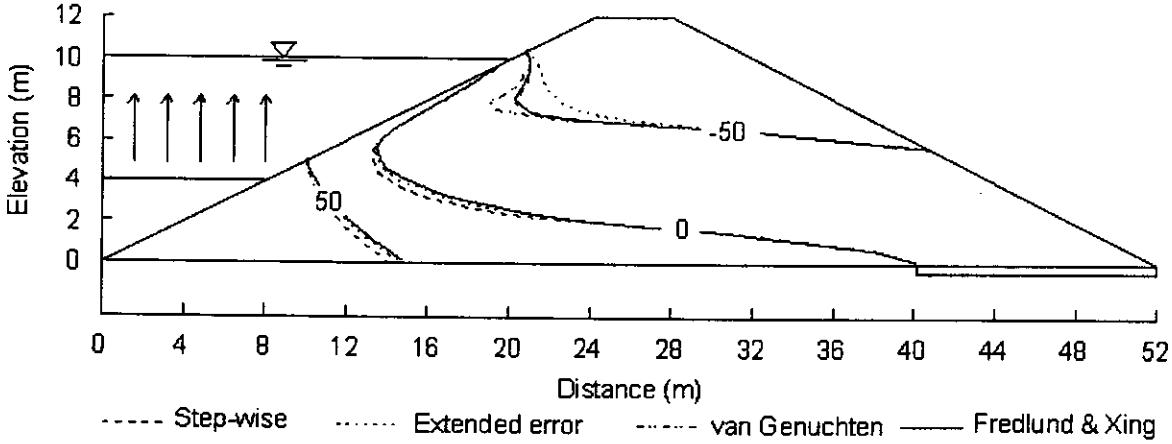

Figure 8. Comparison of the location of pore-water pressure contours for various water storage functions, after an elapsed time, T = 15 hr

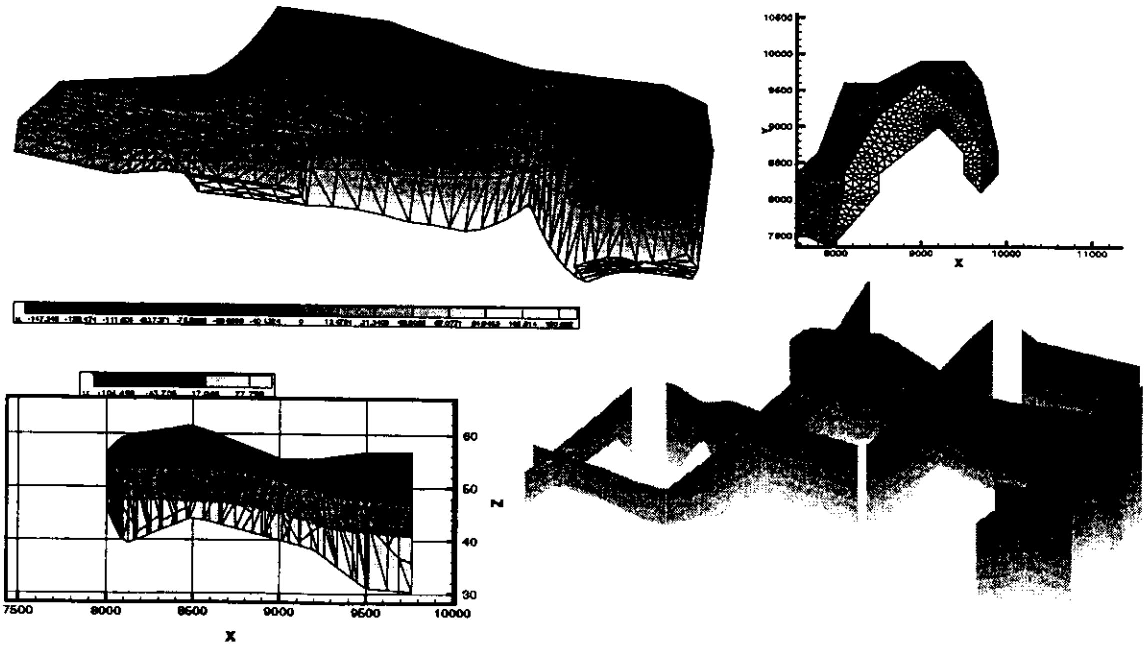

Figure 9. Three-dimensional steady state seepage example (Finite element mesh and the distribution of pressure head; Top view of the finite element mesh; Side view with the distribution of pressure head; and Cross-sections with the distribution of pressure head.

allowing complex geometry to be input based on survey data. Reasonable boundary conditions are then added to the model as well as placing flux sections throughout the problem to model the flow of water. The resulting pressure heads for this problem are presented in Fig. 9

5. CONCLUSION

General partial differential equation solvers and SVFlux appeared to be useful tools for solving saturated-unsaturated seepage problems. These programs are particularly well-suited for solving unsaturated soils problems because of the attention given to: i) ensuring convergence when solving non-linear equations; ii) allowing material properties to be input in a variety of forms; and iii) allowing material properties to be non-linear in character.

It is possible to use a variety of formats for the input of soil property functions. The formats for data input can vary from being a series of data points to a closed-form mathematical equation. In addition, MathCad and SoilVision software can be used in conjunction with SVFlux to compute acceptable mathematical functions for unsaturated soil properties. Database program, SoilVision can be used for the selection of the coefficient of permeability and/or water storage functions for the analysis when experimental data are not available.

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[1] MODFLOW is a proprietary product of U.S. Geological Survey, Virginia, USA

[2] Seep/W is a proprietary product of Geo-Slope International, #1830, 633 –6th Avenue S.W., Calgary, Alberta, T2P 2Y5, Canada.

[3] PDEase2D is a proprietary product of Macsyma Inc., Arlington, MA 02174, USA.

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[9] MathCad is a proprietary product of MathSoft Inc., Dept. 60M40, 101 Main Street, Cambridge, MA 02142, USA