Prediction of the Soil-Water Characteristic Curve from Grain-Size Distribution and Volume-Mass Properties

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ABSTRACT

This paper presents a method of estimating the soil-water characteristic curve from the grain-size distribution curve and volume-mass properties. The grain-size distribution is divided into small groups of uniformly-sized particles. A packing porosity and soil-water characteristic curve is assumed for each group of particles. The incremental soil-water characteristic curves are then summed to produce a final soil-water characteristic curve. Prediction of the soil-water characteristic curve from grain-size distribution allows for a inexpensive description of the behavior of unsaturated soils. The soil-water characteristic curve forms the basis for computer modelling of processes in unsaturated soils.

INTRODUCTION

This paper presents a model for the prediction of the soil-water characteristic curve, (SWCC), based on the particle-size distribution, dry density, void ratio, and specific gravity of a soil. The model first fits a modification of the Fredlund & Xing (1994) equation to the grain-size distribution curve (Figure 1). The grain-size distribution curve is then analyzed as an incremental series of particle sizes from the smallest to the largest in order to build an overall soil-water characteristic curve. Small increments of uniform-sized particles are transposed to obtain a SWCC representing the average particle size. Once the entire grain-size distribution curve is incrementally analyzed, the individual soil-water characteristic curves are superimposed to give the SWCC for the entire soil.

In order to build the general SWCC, it must be assumed that the SWCC for each uniform particle size is relatively unique. Typical soil-water characteristic curves and grain-size distribution curves for a mixture of sand, silt, and clay were obtained from SoilVision (Fredlund, 1996), which contains over 6000 soils. The soil-water characteristic curves were then fitted with the Fredlund & Xing (1994) equation. This provided an approximation for the curve fitting parameters in the Fredlund & Xing (1994) equation classified according to dominant particle size. Parameters used in the Fredlund & Xing (1994) equation for soils composed entirely of sand or entirely of clay are easy to obtain. Uniform soils containing only mid-range particle sizes are more difficult to obtain and as a result some estimation is required.

During development of the algorithm to predict the SWCC, it was decided that provision must be made for the storage of grain-size information. If grain-size information was to be stored, a method of mathematically representing each grain-size curve should be found. The benefits of a mathematical fit would be two-fold. A grain-size curve fit with a mathematical equation would then allow further computations to be performed on the curve. It was reasoned that a prediction of the soil-water characteristic curve would be possible if the grain-size distribution could be fit with an equation. This idea was then implemented in the form of a least-squares curve-fitting algorithm which allowed for fitting of the grain-size distribution data.

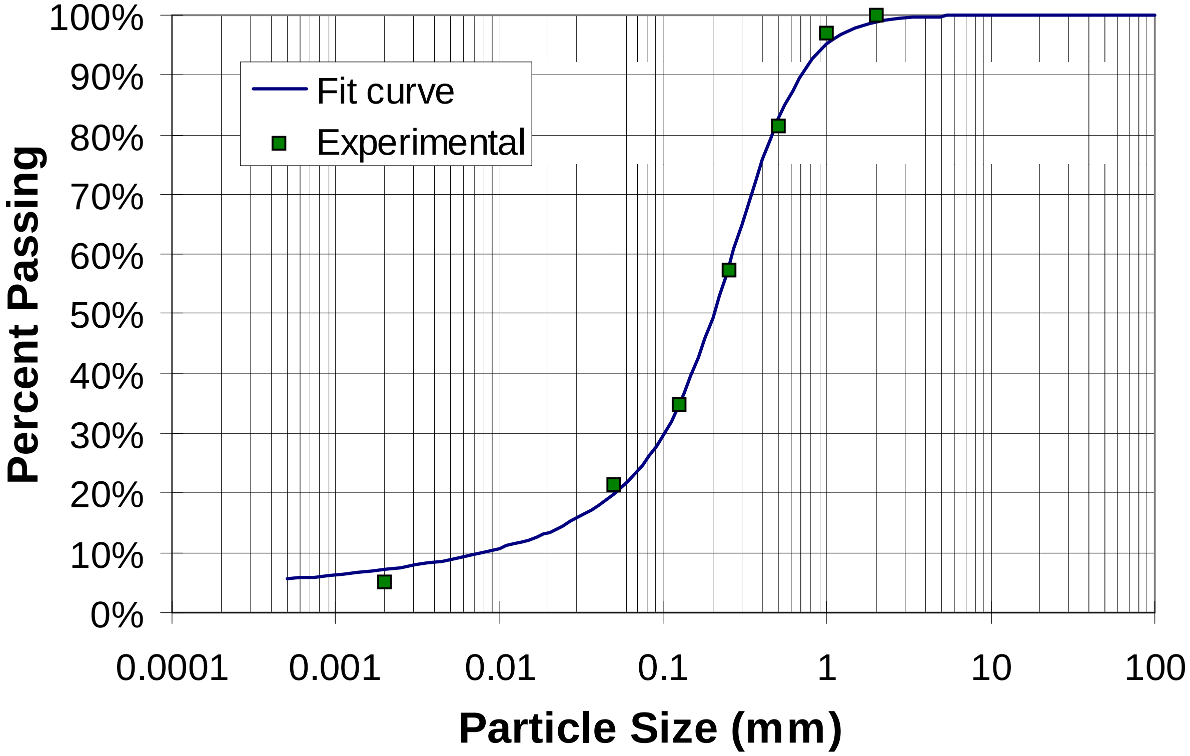


Figure 1 Fit of grain-size curve using a modified Fredlund & Xing (1994) equation (# 10741)

The second benefit of mathematically representing each grain-size curve was that it would provide coefficients of indices by which grain-size curves could be classified. This would allow the ability to search the database for soils with grain-size curves within a specified band. This technique has proven invaluable in performing sensitivity analyses on soil parameters.

THEORY FOR MATHEMATICALLY REPRESENTING THE GRAIN-SIZE DISTRIBUTION CURVE

Previous research work to fit the grain-size curves was reviewed (Wagner, 1994). The research by Wagner presented several lognormal distributions capable of fitting the grain-size curve. Providing a meaningful representation of the grain-size data in the extremes proved difficult for a lognormal distribution.

Due to similarity between the shape of the grain-size distribution and the shape of the soil-water characteristic curve, a different approach was taken. The Fredlund & Xing (1994) equation, which had previously been used to fit SWCC data, provided a flexible and continuous equation that could be fit by the nonlinear regression using three parameters. The equation was modified to permit the fitting of grain-size curves. The modified equation, [0.1], allowed for a continuous fit and proper definition of the extremes of the curve.

$$P_p(d) = \frac{1}{\ln\left[\exp(1) + \left(\frac{g_a}{d}\right)^{g_n}\right]^{g_m}} \left[1 - \left[\frac{\ln\left(1 + \frac{d_r}{d}\right)}{\ln\left(1 + \frac{d_r}{d_m}\right)}\right]^7\right] \qquad \textbf{[0.1]}$$

where:

$P_p(d)$ = percent passing a particular grain-size, d

g_a = fitting parameter corresponding to the initial break in the grain-size curve,

g_n = fitting parameter corresponding to the maximum slope of grain-size curve,

g_m = fitting parameter corresponding to the curvature of the grain-size curve,

d = particle diameter (mm),

d_r = residual particle diameter (mm),

d_m = minimum particle diameter (mm)

THEORY OF PREDICTING THE SOIL-WATER CHARACTERISTIC CURVE FROM THE GRAIN-SIZE DISTRIBUTION

The mathematical fit of the grain-size distribution led to the development of an algorithm capable of predicting the soil-water characteristic curve. A review of current research showed that one of two approaches have typically been taken in the prediction of the soil-water characteristic curve from grain-size. The first approach entails a statistical estimation of properties describing the SWCC from grain-size and volume-mass properties (Gupta, 1979; Ahuja, 1985; Ghosh; 1980; Aberg, 1996) It appeared that a theoretical approach to the problem would hopefully provide superior predictions.

The second approach was theoretical and involved converting the grain-size distribution to a pore-size distribution which was then developed into a SWCC (Arya, 1981). This research was duplicated and compared to experimental data. Difficulty was encountered in generating a reasonable SWCC along the entire range. Predicted soil-water characteristic curves typically showed abnormal “humps” and fell to zero volumetric water content long before the experimental data was completely desaturated (Figure 2).

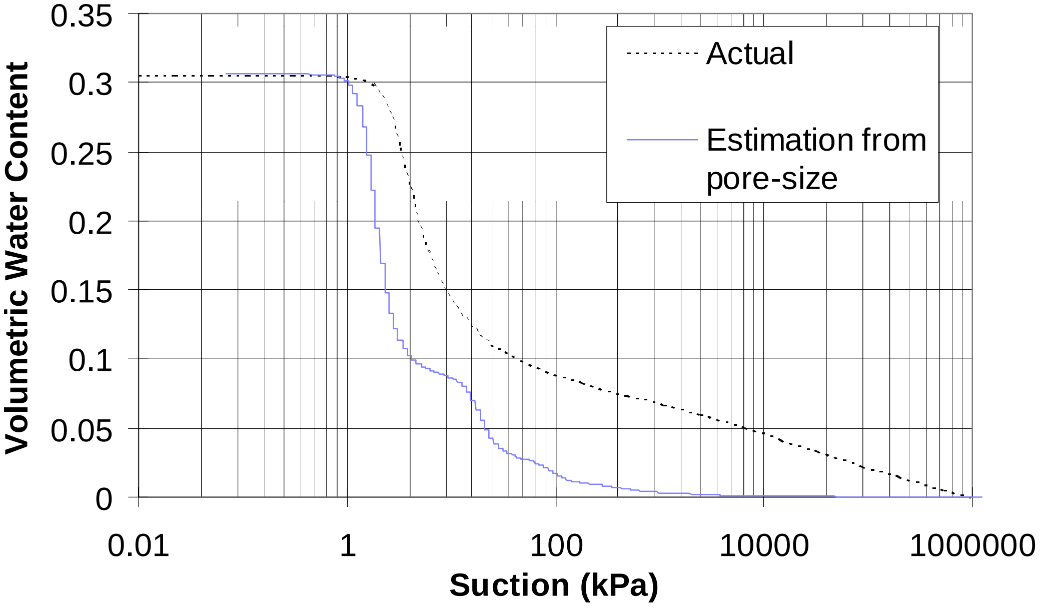


Figure 2 Illustration of abnormalities associated with prediction of SWCC from pore-size distribution

A new approach is proposed for predicting the soil-water characteristic curve from the grain-size distribution curve. It was assumed that a soil composed entirely of a uniform, homogeneous particle size would have a unique soil-water characteristic curve. The shape of the SWCC for pure sands, pure silts and pure clays was known. Using a best-fit analysis with the Fredlund & Xing (1994) equation, three parameters were computed for each soil type. It was then assumed that these parameters could be associated with a dominant particle size on the grain-size plot. The uniqueness of the soil parameters was confirmed by querying the SoilVision database for plots of the 'n' and 'm' parameters versus the percent sand, silt, and clay of a soil. It was hypothesized that as a soil tended towards uniformity, the 'n' and 'm' parameters would show a trend towards a particular value. The particle sizes falling between pure clays, pure silts and pure sands were then approximated.

This resulted in the production of two plots, one for the 'n' parameter, and one for the 'm' parameter. These plots described the variation in the 'n' and 'm' parameters with grain-size. This allowed n and m parameters to be estimated for any soil composed of uniform diameter particles.

The grain-size distribution curve can be divided up into small divisions of uniform soil particles. Starting at the smallest diameter size, a packing porosity was estimated (Harr, 1977) for each division and a soil-water characteristic curve estimated as shown in Figure 3. The divisional soil-water characteristic curves can then be summed starting with the smallest particle size and continuing until the volume of pore space is equal to that of the entire heterogeneous soil. The result is a theoretically predicted soil-water characteristic curve.

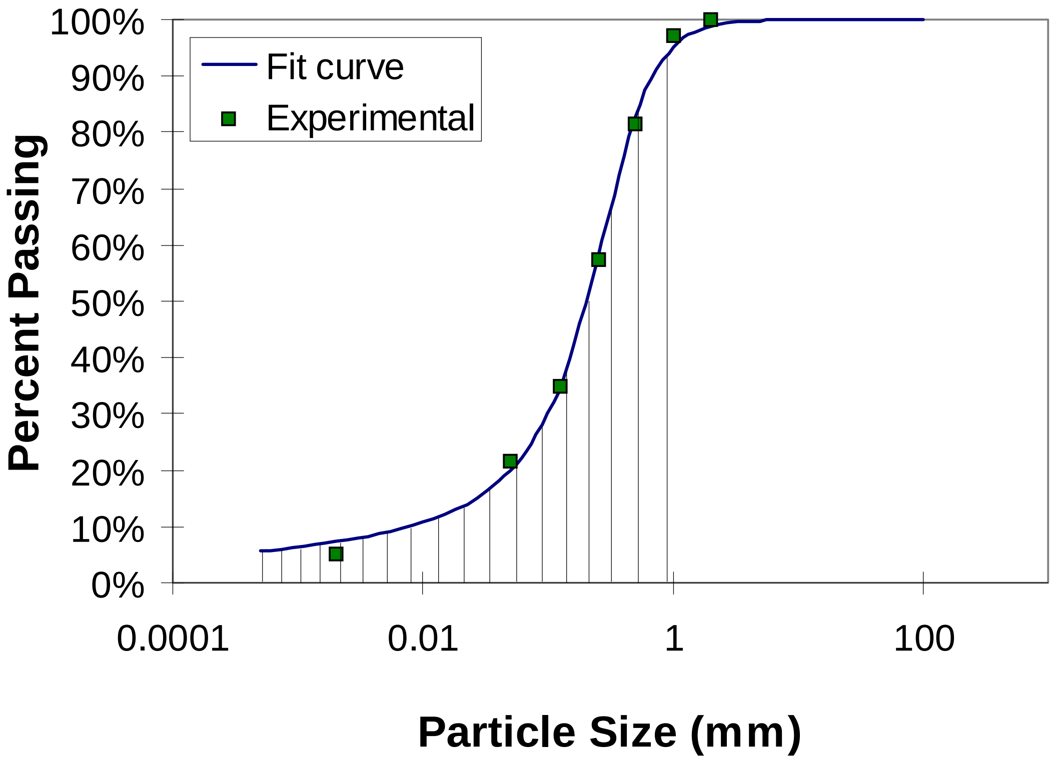
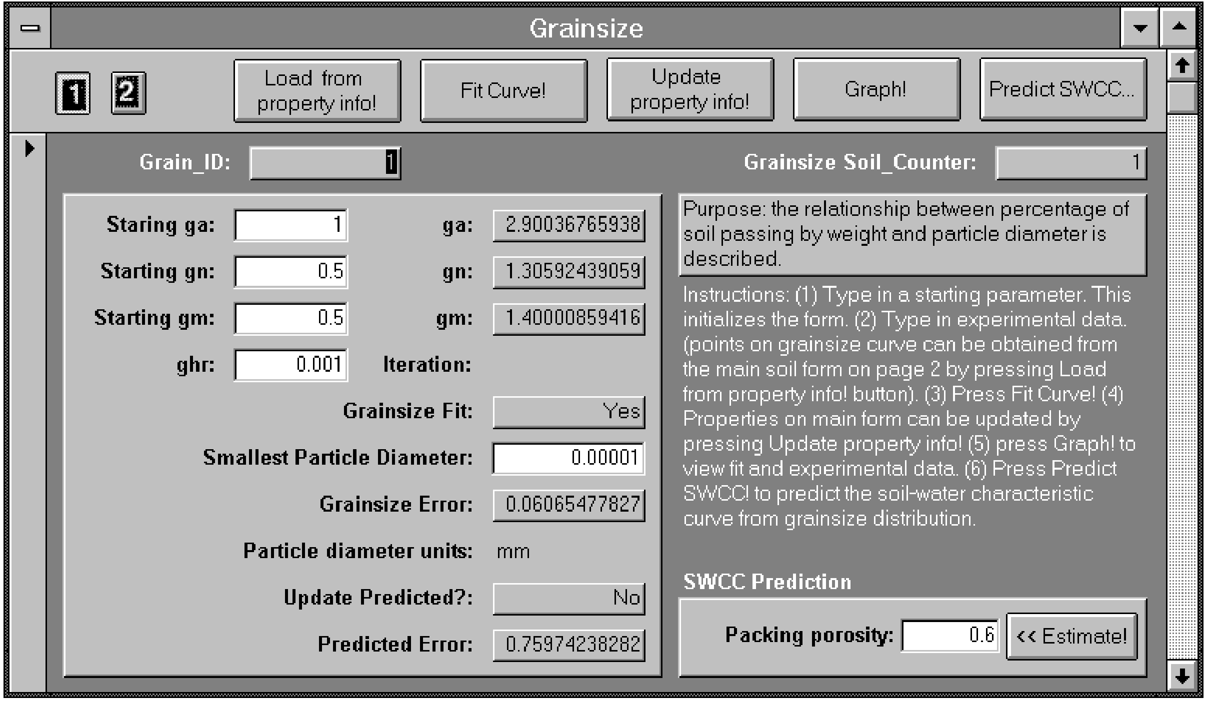


Figure 3 Small divisions of particle size used to build complete SWCC

IMPLEMENTATION OF THE SWCC PREDICTION INTO SOILVISION

Information relevant to describing the grain-size distribution is organized in a single form in the SoilVision knowledge-based system. Figure 4 shows the grain-size form for the knowledge-based system. Two pages are required to present the information. The first page contains parameters controlling the fit of grain-size, the smallest particle diameter, the error between the fit data and experimental data, the error between predicted SWCC and experimental data predicted and experimental data, and counters which Access® uses to identify individual records. Page one also contains the packing porosity field which controls the prediction of the soil-water characteristic curve. Page two displays the equation used to fit the experimental data as well as experimental data points and generated data points on the best fit curve. Figure 5 shows the second page of the grain-size distribution form.

The header on the form allows for a number of helpful functions and algorithms. If soil data consists of % Coarse, % Sand, % Silt, %Clay or D10, D20, D30, D50, or D60 data on page two of the main soil form, pressing a button will convert this data into experimental points along the grain-size distribution graph. Once experimental data is obtained, pressing Fit Curve! will initiate the linear regression algorithm that will best-fit the equation to experimental data. The results of the fit can be viewed by pressing the Graph! button and a soil-water characteristic curve can be predicted by pressing the Predict SWCC... button.



Grain_ID: 1

Grainsize Soil_Counter: 1

Staring ga:	1	ga:	2.90036765938
Starting gn:	0.5	gn:	1.30592439059
Starting gm:	0.5	gm:	1.40000859416
ghr:	0.001	Iteration:	

Grainsize Fit: Yes

Smallest Particle Diameter: 0.00001

Grainsize Error: 0.06065477827

Particle diameter units: mm

Update Predicted?: No

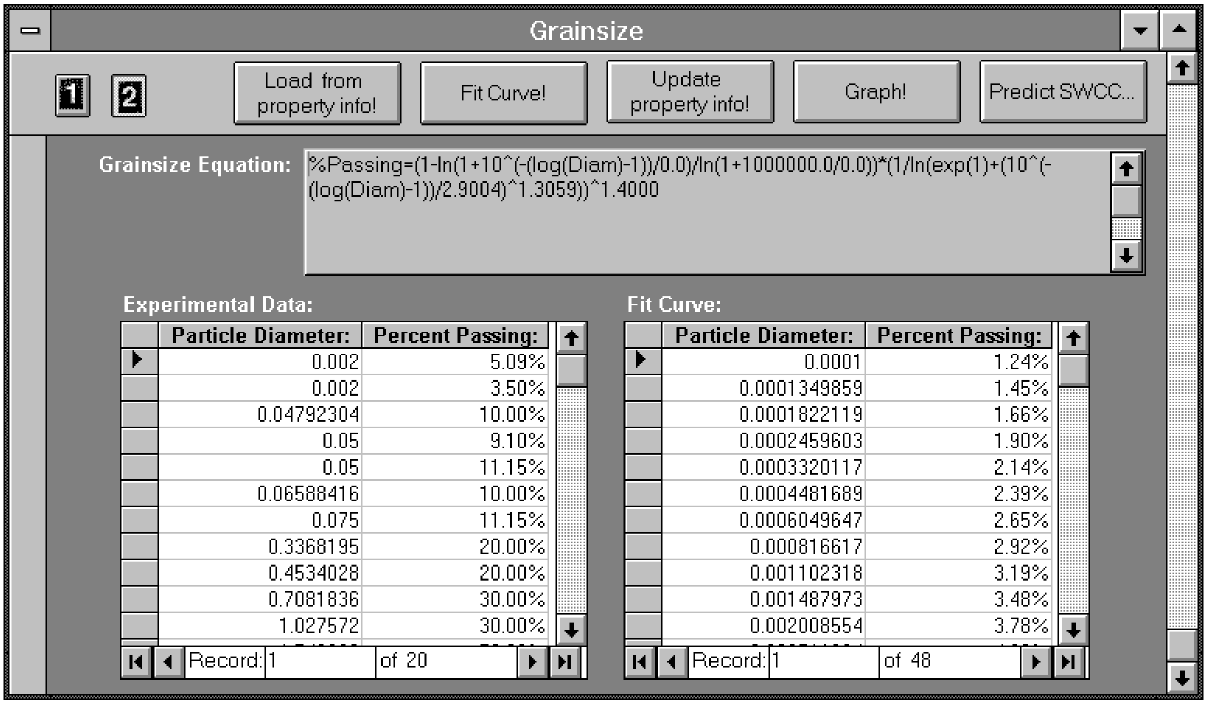
Predicted Error: 0.75974238282

Purpose: the relationship between percentage of soil passing by weight and particle diameter is described.

Instructions: (1) Type in a starting parameter. This initializes the form. (2) Type in experimental data. (points on grainsize curve can be obtained from the main soil form on page 2 by pressing Load from property info! button). (3) Press Fit Curve! (4) Properties on main form can be updated by pressing Update property info! (5) press Graph! to view fit and experimental data. (6) Press Predict SWCC! to predict the soil-water characteristic curve from grainsize distribution.

Packing porosity: 0.6

Figure 4 Page one of the grain-size distribution form



Grainsize Equation: %Passing=(1-ln(1+10^(-(log(Diam)-1))/0.0)/ln(1+1000000.0/0.0))*(1/ln(exp(1)+(10^(-(log(Diam)-1))/2.9004)^1.3059))^1.4000

Experimental Data:

Particle Diameter:	Percent Passing:
0.002	5.09%
0.002	3.50%
0.04792304	10.00%
0.05	9.10%
0.05	11.15%
0.06588416	10.00%
0.075	11.15%
0.3368195	20.00%
0.4534028	20.00%
0.7081836	30.00%
1.027572	30.00%

Record: 1 of 20

Fit Curve:

Particle Diameter:	Percent Passing:
0.0001	1.24%
0.0001349859	1.45%
0.0001822119	1.66%
0.0002459603	1.90%
0.0003320117	2.14%
0.0004481689	2.39%
0.0006049647	2.65%
0.000816617	2.92%
0.001102318	3.19%
0.001487973	3.48%
0.002008554	3.78%

Record: 1 of 48

Figure 5 Page two of the grain-size distribution form

CONCLUSIONS

The readapted Fredlund & Xing (1994) equation produces a satisfactory fit of the grain-size distribution. Figure 3 shows that the experimental data can be fit with a minimal error. A good curve fit of the grain-size curve is essential for the prediction of a reasonable soil-water characteristic curve. The minimum particle size was also found to have an influence on the prediction of the soil-water characteristic curve prediction. If the minimum particle size variable was too low, the overabundance of clay size particles woul dominate the prediction. If the minimum particle size was too high, an absence of smaller particles would result in the soil drying out prematurely.

The prediction of soil-water characteristic curve from the grain-size distribution was found to be particularly accurate for sands, and reasonably accurate for silts. Clays, tills and loams were more difficult to predict although the accuracy of the prediction algorithm appears to be reasonable. Results tended to be sensitive to the packing porosity and more research is required in this regard. Soils with experimental data for both the grain-size curve and the soil-water characteristic curve were extracted from the database. The results of comparisons between experimental and predicted data can be seen in Figure 6, Figure 7, Figure 8, Figure 9, Figure 10, Figure 11, Figure 12, and Figure 13.

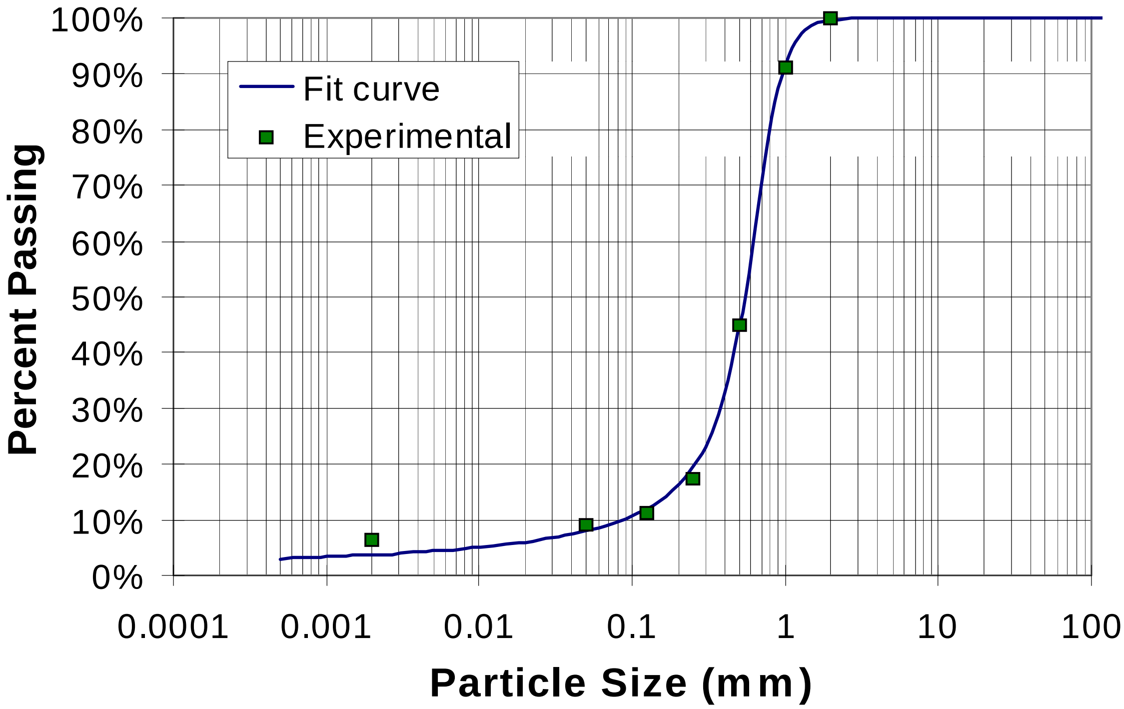


Figure 6 Grain-size distribution fit for a Sand (# 10720)

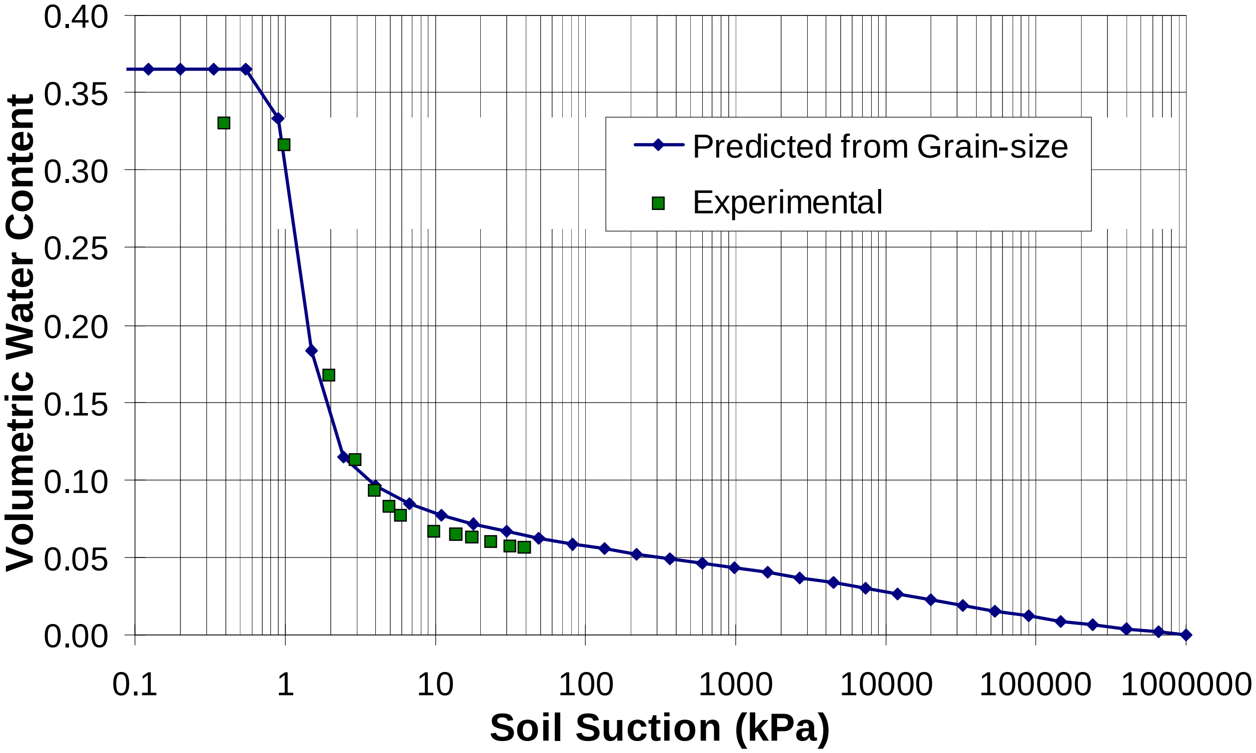


Figure 7 Comparison between experimental and predicted curves for Sand (# 10720)

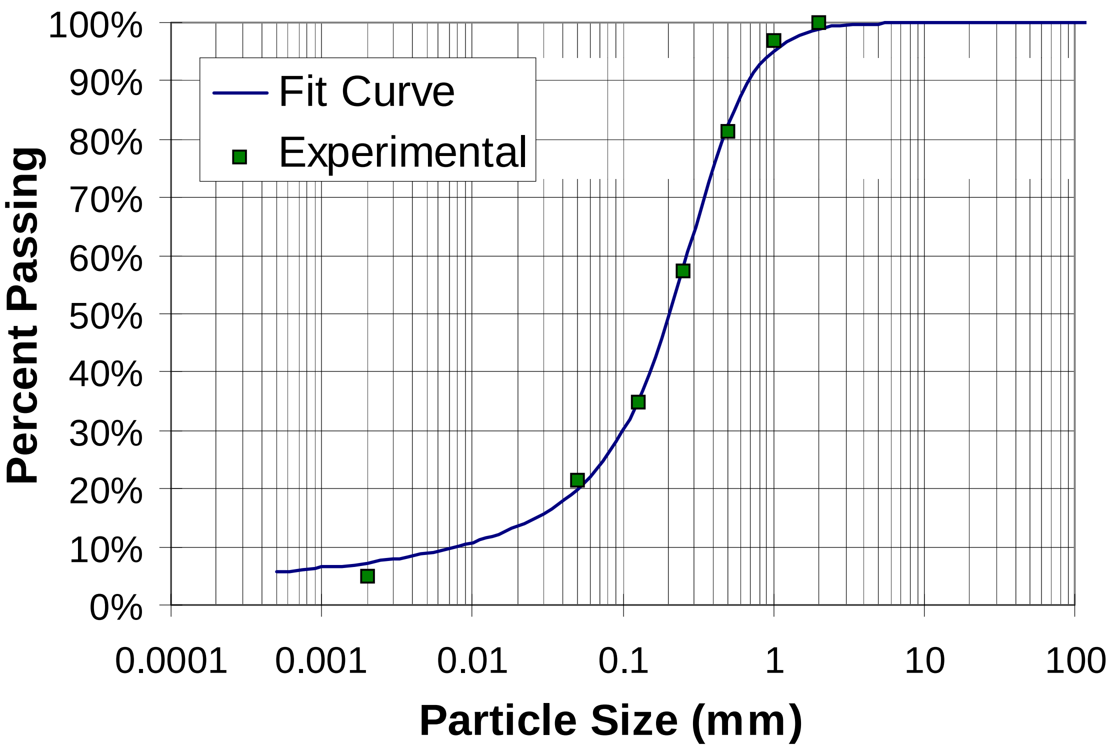


Figure 8 Grain-size distribution fit for a Loamy Sand (# 10741)

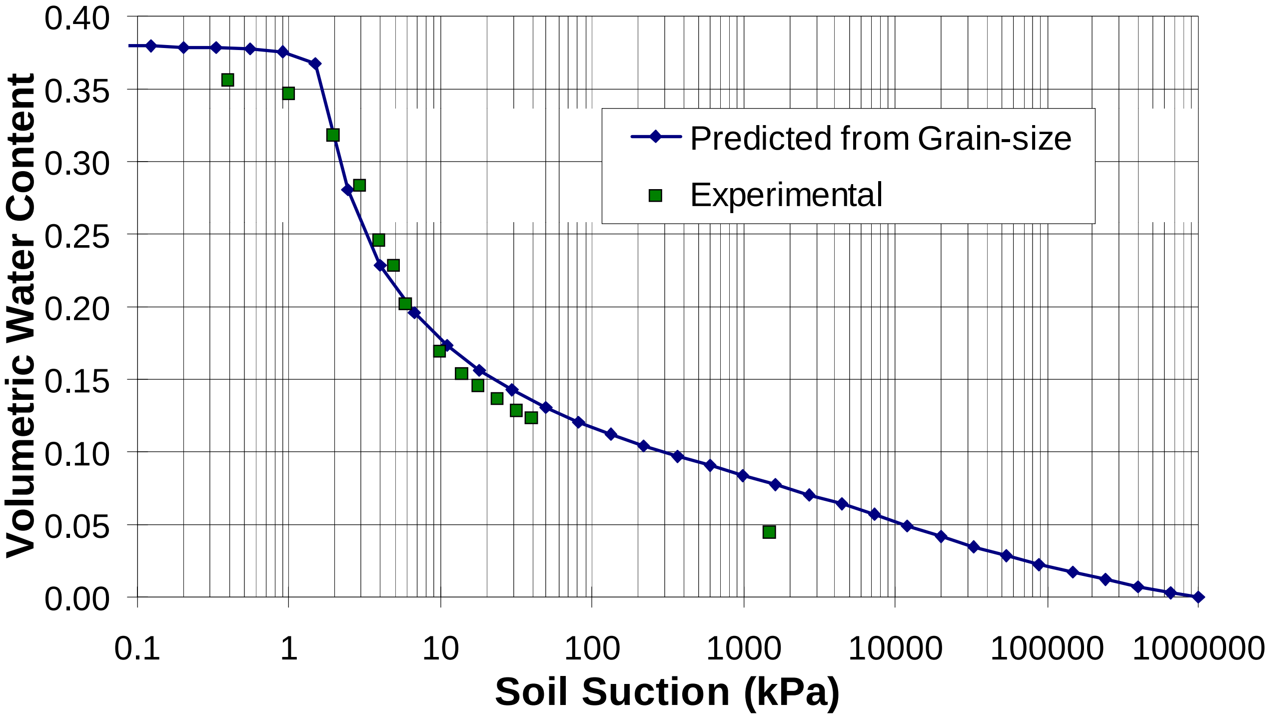


Figure 9 Comparison between experimental and predicted curves for a Loamy Sand (# 10702)

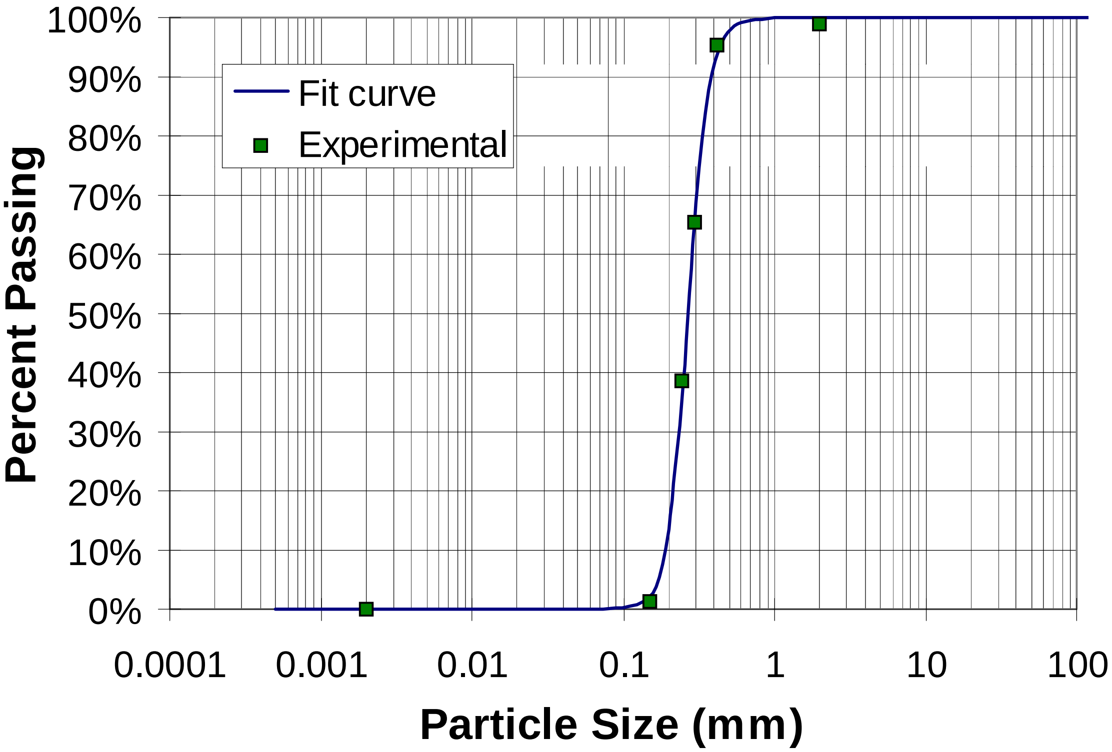


Figure 10 Grain-size distribution for a Sand (# 350)

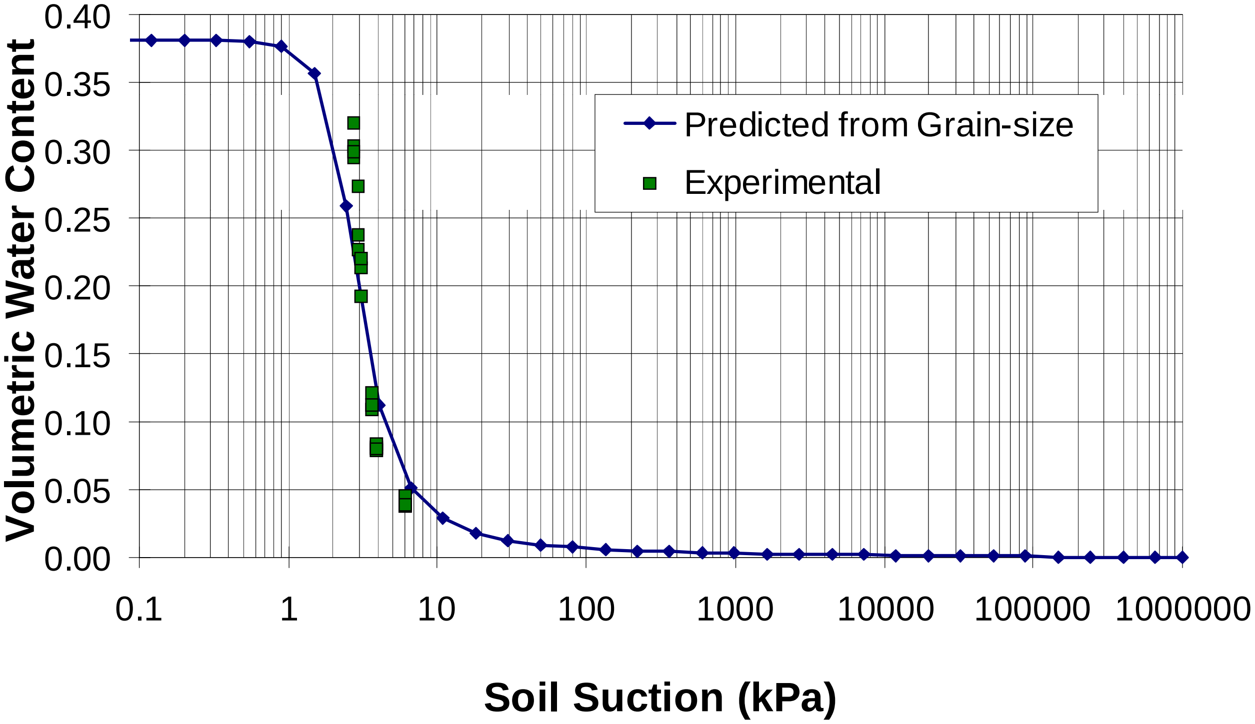


Figure 11 Comparison between experimental and predicted data for a Sand (# 350)

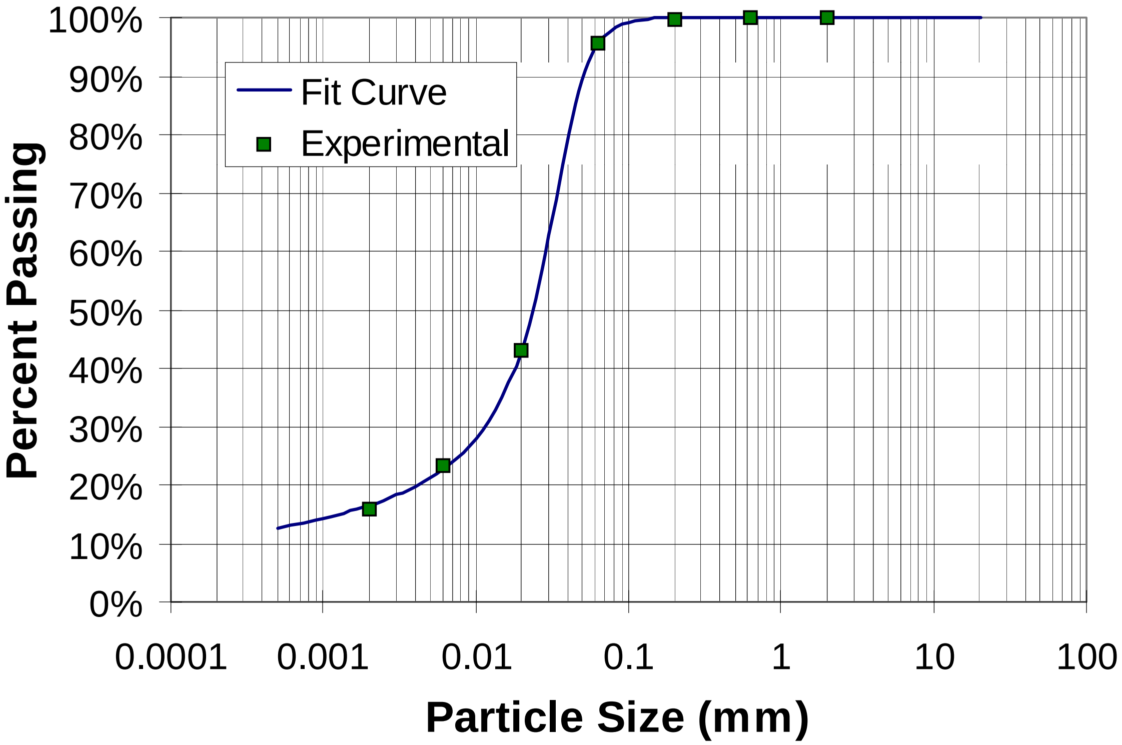


Figure 12 Grain-size distribution for a Silt Loam (# 10861)

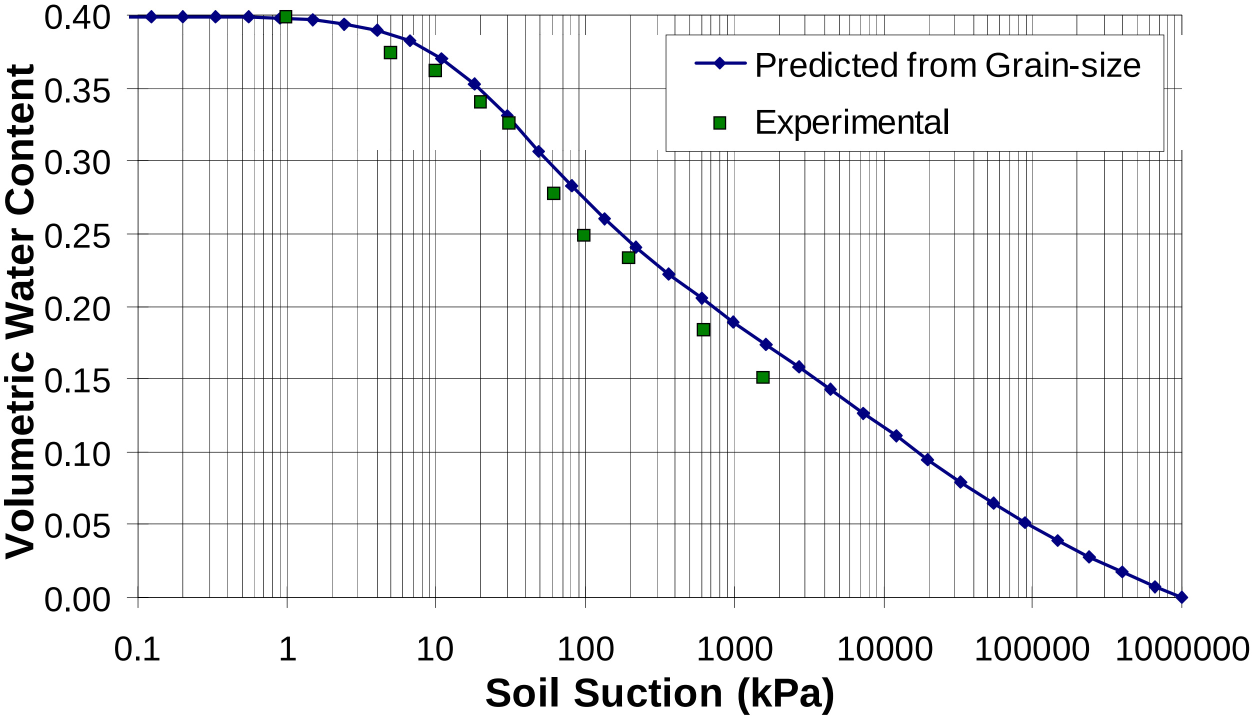


Figure 13 Comparison between experimental and predicted data for a Silt Loam (# 10861)

ACKNOWLEDGEMENTS

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