From experimental evidence towards the assessment of weather-related railway embankment hazards

Gilson Gitirana Jr, Ph.D. student
Delwyn G. Fredlund, Professor Emeritus

Department of Civil and Geological Engineering, University of Saskatchewan, 57 Campus Drive, Saskatoon, SK, Canada, S7N 5A9

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ABSTRACT
Weather-related geo-hazards are a major concern for the railway industry in Canada. The financial losses that result from derailments and delays amount to millions of dollars every year. On the other hand, the assessment and management of geo-hazards is a difficult problem that involves complex coupled phenomena and numerous soil and weather parameters. The primary goal of this paper is to illustrate the manner whereby unsaturated soil mechanics can be taken from the soil property assessment level using techniques based on the soil-water characteristic curve (SWCC), to the solution of this real and highly complex problem.

First, a concise description of the weather-related geo-hazards assessment model (W-GHA model) is given. Deterministic and probabilistic aspects of the model were developed within a Decision Analysis framework. The deterministic core of the model consists of a two-dimensional stability analysis combined with the analysis of the effects of weather conditions on the pore-water pressures. According to the proposed model, weather conditions interact with the ground through the flow of liquid water, water vapour, and heat. Critical embankment stability conditions are determined using a Dynamic Programming Method (DPM) combined with Finite Element based stress fields. The soil system is ultimately represented by a series of partial differential equations (PDE’s) governing conservation of mass and momentum. A discrete stochastic analysis is implemented within the proposed framework.

Several unsaturated soil property functions are required as input to the system of PDE’s. The hydraulic conductivity (i.e., coefficient of permeability), vapour diffusion coefficient, thermal conductivity, volumetric specific heat, and shear strength are all nonlinear functions physically related to the SWCC. The methodology by which these soil property functions can be inter-related to the SWCC is presented. The theoretical model illustrates the manner whereby it is possible to quantitatively assess embankment stability based on weather conditions. The methodology is feasible and yet relatively comprehensive.
1 INTRODUCTION

This paper presents theoretical and practical aspects associated with a weather-related geo-hazard assessment model (W-GHA model) developed for railway embankments. The study was carried out taking into account the specific needs of the Canadian Pacific Railway (CPR). However, the W-GHA model should be applicable to the railway industry in general. Special focus is given to the saturated/unsaturated soil modelling and the assessment of unsaturated soil property functions. First, a description of the problem background is given, explaining the decision situation the railway industry faces. A description of how the railway could benefit from a geo-hazard assessment model is given, making the case for the potential benefits of using unsaturated soil mechanics to assess weather-related geo-hazards. Secondly, a description of the Decision Analysis framework for railways is given, embracing the deterministic and probabilistic features of the model. The following section presents the W-GHA model, describing the unsaturated soil theories required to assess weather-related geo-hazards in addition to explaining the required weather and soil parameters. The method of assessing the unsaturated soil properties is described, emphasising the central role played by the soil-water characteristic curve, SWCC. Finally, a framework for frequency and sensitivity analyses is provided.

1.1 Problem background

Geo-hazards create a serious impact on the performance of the Canadian railway networks. The most important impact involves a compromise of deadlines, an expense for the repair of railway track and damaged sites, and the safety exposure for employees and the public. As a result, the minimization of geo-hazards impacts is an issue of great concern for the railways. The railway industry faces a serious dilemma; namely, the industry operates networks with tens of thousands of kilometres of line, crossing several types of geographical terrain, soil, and weather conditions from coast to coast (see the CPR network on Fig. 1). It is extremely difficult to protect and/or remediate every site under significant risk. At the same time, risks must be managed in an affordable manner.

The importance of geo-hazards is illustrated through derailment statistics. Figure 2 presents the number of main-track derailments reported by the Canadian railways to the Transportation Safety Board of Canada (TSB 1994) combined with data from the following years (TSB 2001). The ‘main track’ is defined as a track extending through yards and between stations whereas ‘non-main track’ is composed mostly of sidelines and yards. Figure 2 shows that Canadian main-track derailments have declined by a factor of almost three between 1980 and 1988, followed by a roughly level trend thereafter, despite a peak of derailments in 1996. A series of equipment improvement initiatives undertaken during the 1980's were essentially exhausted after 1988, explaining in part why derailments have levelled or even increased thereafter.
Fig. 1. Canadian Pacific Railway network and main landslide areas according to the type of terrain (terrain data from Office of Critical Infrastructure Protection and Emergency Preparedness 2001).

Fig. 2. Main-track derailment data provided by the Transportation Safety Board of Canada (1994 and 2001).

In 2001, the Transportation Safety Board provided more detailed rail occurrence statistics, including non-main track data. Figure 3 presents the total number of derailments classified by assigned factors, along with the number of fatalities and injuries caused by derailments from 1992 to 2001. Geometry and roadbed factors are closely related to geo-hazards whereas equipment factors are associated to mechanical failures and defects. Action factors are related to inappropriate
train and track operation. Figure 3 shows that the number of fatalities and injuries caused by derailments can be considered small when compared to the numbers of fatalities and injuries associated with crossing and trespasser accidents, which reaches the hundreds annually. A high number of derailments suggest a potentially high financial impact. Clear trends for the entire period from 1992 to 2001 are non-existent, but the number of derailments is relatively constant as shown in Fig. 3. One important observation shown by the data is that the cross-section geometry and roadbed factors have a significant contribution to the number of derailments, demonstrating the importance of geo-hazards for railway safety.

1.2 Geo-hazards affecting Canadian railways

The geo-hazards affecting the Canadian railway networks can be classified into five categories; namely, debris flows, embankment failures, rock falls, volume change (subsidence and frost heave), and erosion washouts. Figure 4 groups together the first four categories as “slope and subgrade failure and/or serviceability” problems. Most geo-hazards are clearly a direct function of weather conditions and are usually triggered by severe weather conditions/storms. Embankment failures and debris flows represent relatively low frequency and high impact hazards, while other categories have higher frequencies but a lower impact.

In the past, geo-hazards have received the same emphasis as equipment-related hazards, but there has been less success in dealing with geo-hazards due to the complexity of assessing ground conditions. Improvements in real-time hazard detection systems and the implementation of track inspection programs have somewhat improved the situation. Detection instrumentation systems are effective but are an expensive option that still requires a great deal of technological improvement. Inspection programs only partially assist in identifying track geometry problems because washouts and slides can quickly cause roadbed deterioration. In order to adequately identify geometry related problems, an economically prohibitive number of inspection teams would have to be dispatched.

Improvements in geo-hazard assessment techniques can assist in indicating appropriate inspections frequencies, thereby providing a way to rationalize railway resources. CPR has implemented slope hazard assessment and management programs. The programs are showing promise but limited results are available. One such example is the rock slope program implemented during the mid 1970's, using a qualitative priority rating system (ranking method). While qualitative assessment methods satisfy many practical needs, the advantages of quantitative methods cannot be ignored. Morgenstern (1997) presents a list of advantages of quantitative assessment approaches. Mackay (1997) showed that the CPR hazard management programs would greatly benefit from a procedure for quantitative assessment. Most geo-hazards are triggered by severe weather conditions, and therefore a quantitative hazard assessment method meeting the needs of CPR should involve quantification of the influence of weather. The quantification of weather effects can be done through a soil-atmospheric moisture exchange model (Wilson et al. 1994).
**Fig. 3.** Main- and non-main track derailments by assigned factors and injuries/fatalities due to derailments (data from Transportation Safety Board of Canada 2001).

**Fig. 4.** Categorization of geo-hazards.

The framework for the assessment of weather-related geo-hazards under study will be limited to embankment ruptures. The general framework should also be applicable for the assessment of the other categories of geo-hazards. Infiltration and exfiltration are considered in this paper, but the influences of freezing and thawing processes are not considered.
1.3 Soil-water characteristic curve as a hazard gauge

Embankment hazards are strongly related to the reduction in soil suction within the embankment. Soil suction varies according to the amount of water stored within the embankment. On the other hand, changes in the amount of water within the soil are associated with the weather conditions and soil properties. The relationship between the amount of water being stored in the soil and soil suction is given by the soil-water characteristic curve, SWCC (Figure 5). Typical unimodal SWCC’s can be defined using four soil parameters; namely, the air-entry value, $\psi_b$, the residual suction, $\psi_{res}$, the residual degree of saturation, $S_{res}$, and a parameter defining the sharpness of the transitions at the bending points, ‘$a$’ (Gitrana Jr. & Fredlund 2003). The shape of the SWCC is a function of the pore-size distribution in the soil, amongst other factors.

Figure 6 presents an analogy, explaining the fundamental concept behind the W-GHA model. According to this analogy, the soil comprising an embankment can be viewed as a ‘water tank’. The SWCC works as a gauge, an indication of the water level within the ‘water tank’. The water level is lowered through evaporation and downward seepage and raised through infiltration from rainfall. The embankment factor of safety, $F_s$, and the embankment hazard level vary according to the water level. A low water level corresponds to a lower level of hazard (higher $F_s$), while higher water levels produce greater hazards (lower $F_s$). The rigorous model for the assessment of embankment hazards presented herein is based on the ‘water tank’ concept and on the factor of safety. However, it is also possible that a more crude approach could rely directly on the water level within the ‘water tank’. Such an approach would still capture the main factors controlling the stability of railway embankments.

![Soil-water characteristic curve conceptualization](image)

Fig. 5. Soil-water characteristic curve conceptualization.
Fig. 6. SWCC as a water level and embankment hazard gauge.

2 DECISION ANALYSIS MODEL FOR WEATHER-RELATED GEO-HAZARDS

The W-GHA model is composed of deterministic and probabilistic elements that are developed within a Decision Analysis framework. Einstein et al. (1978) and Einstein (1997) present some applications of the Decision Analysis technique to geotechnical engineering problems. According to the definition given by Keeney and Raiffa (1976), Decision Analysis provides a structure for systematically analysing difficult situations. Bunn (1984) emphasized that Decision Analysis should not be viewed as a substitute for the personal judgement of decision makers, but as a complement. Bunn’s observations are similar to the views of several researchers with regard to the role of probabilistic analysis in geotechnical engineering. These authors recommend the use of a reliability-based analysis not as a substitute, but along with the past experience accumulated after generations of geotechnical engineering (Whitman 1984, Fell 1994, and Morgenstern 1995).

According to Clemen (1996), there are four basic sources of difficulty in decision analysis. These are: (1) problem complexity; (2) inherent uncertainty of the situation; (3) decisions involving multiple objectives; and (4) problems where different perspectives lead to different conclusions. The W-GHA model is a complex, multidisciplinary model that brings together concepts of geotechnical and hydrological engineering for a complex problem. There are also inherent uncertainties due to soil properties and weather conditions. The third source of difficulty listed above can be considered to be ‘not important’ if the analysis of geo-hazards is kept sufficiently narrow.
Fig. 7. A Decision Analysis process cycle (modified from Clemen 1996).

All railway objectives presented in the next section depend on the minimization of hazards. The minimization of hazards can be expressed in terms of the single objective to maximize the factor of safety for the problem at hand. The fourth type of difficulty listed above should be of minor importance when compared to the previous sources of difficulty, as long as broadly accepted geotechnical engineering approaches are undertaken.

The Decision Analysis flowchart has seven steps (Fig. 7). The first two steps in the Decision Analysis cycle, ‘identification of decision situations’ and ‘identification of alternatives’ are presented in this section. Step number three, ‘decomposition and modelling’ is presented later. Systematic criteria for the selection of best alternatives and systematic steps in a sensitivity analysis can be learned from the Decision Analysis approach. The choice of a best alternative can be temporarily eliminated and the problem reduced to a reliability-based analysis problem.

2.1 Identification of railway objectives and means

Clemen (1996) offers guidelines for the identification of objectives and means in Decision Analysis. This portion of the decision modelling process is often referred as ‘brainstorming’. Figure 8 describes the fundamental objectives hierarchy for the railway, for the context considered herein. The maximization of the railway system performance can be considered to be the most fundamental objec-
tive. Minimization of financial losses, injuries, loss of life, and the release of dangerous goods are objectives to be reached as part of the most fundamental objective. Finally, the objectives in the lower hierarchical places are the minimization of all types of geo-hazards. The hazard may be a mechanical, geotechnical, or a human action problem. The widely used global factor of safety concept will be used to quantify embankment stability hazard levels. Debris flows, rock falls, and volume change problems may require different hazard quantification parameters.

The identification of the means available to deal with geo-hazards is less straightforward. Figure 9 presents the means-objectives network for the railway system. The means to maximize the railway system performance are numerous, and only a few are presented in the figure in order to keep the context of the problem from becoming excessive. Two distinct types of means can be identified; namely, reactive and proactive means. As geo-hazard assessment techniques were not available in the past, the rail industry needed to invest most of its resources on reactive methodologies. As our knowledge and understanding of geo-hazards develops, new proactive methodologies become an option. Figure 9 presents the most important means towards the implementation of proactive methodologies.

2.2 Identification of railway alternatives

The decision at hand has two possible alternatives; namely, to take action or not take action. The question of whether or not action needs to be taken is of paramount importance in the management of a railway system. The ability to identify high-level hazards along the railway would render considerable savings. By taking precautionary measures, loss of equipment can be prevented and the safety of the railway workers increases, reducing the number of injuries and the chance of fatalities. A major concern to the railway companies is the frequent delays caused by accidents along the track, and in this case delays could be reduced.

![Diagram](image)

**Fig. 8.** A Fundamental objectives hierarchy of the W-GHA model.
Maximize railway system performance

Prevent and/or protect against failures in advance (proactive)
- Improve weather forecast
- Improve ground and hazard characterisation
- Improve analytical models
- Improve computational tools and methods
- Develop database systems

Improve response and/or remediation measures (reactive)
- Develop and/or obtain better equipment
- Training of technical and field personnel
- Improve field procedures

Fig. 9. Means-objectives network of the W-GHA model.

The two generic alternatives (i.e., to take action or not take action) are considered herein. However, different hazard levels and conditions would require different actions. Stronger actions must be taken for greater risks and different actions must be taken for different hazard characteristics. Details regarding the railway system management and the consideration of types of actions to be taken are issues that must be addressed by the Decision Makers. Specific alternatives can be considered using the proposed framework.

3 Model for the problem structure

The deterministic modelling of railway embankment stability problems represents the most complex part of developing the W-GHA model. A traditional geotechnical engineering approach is used in the sense that the stability of railway embankments is idealized as a function of the stress state (i.e., total stresses and pore-water pressure) and the shear strength along a critical slip surface within the earth mass. The Factor of Safety, $F_s$, is determined using an optimization technique.

The W-GHA model is based on a series of partial differential equations governing the thermo-hydro-mechanical behaviour of the saturated/unsaturated soil system. Appropriate boundary conditions to account for evaporation, precipitation, and runoff are given. The manner in which the pore-water pressures and total stress distributions are used to determine stability is explained later.
3.1 W-GHA model components

Figure 10 illustrates the environmental factors affecting the stability of an embankment. Changes in the stress state distribution and shear strength within the soil mass take place in response to moisture fluxes at the soil-atmosphere boundary. In order to determine the moisture flux and changes in pore-water pressure in the soil, partial differential equations (PDE’s) governing the flow of water must be combined with appropriate boundary conditions and solved for the period of time under consideration. The PDE governing the flow of heat must also be solved since the amount of liquid water and water vapour flow depends on the temperature, which in turn, changes in response to the energy available at ground surface. Special procedures to determine the amount of run-off and actual evaporation are also required. The total stress distribution and the shear stress acting along a particular slip surface are obtained by solving the PDE’s governing static equilibrium of forces.

The W-GHA model incorporates the influence of soil-atmosphere moisture fluxes on the stability of an embankment according to the flowchart presented in Figure 11. An essential component of the model is the Dynamic Programming algorithm associated with the slope stability analysis. The algorithm is used to determine critical conditions and the corresponding factors of safety $F_s$ at any time, $t$. In order to determine critical conditions, the net stress distribution at $t = t_i$ and the corresponding pore-water pressure distribution, $u_{wp}$, at $t = t_i$ are required. The pore-water pressure distribution is determined by solving two-dimensional, partial differential equations governing transient moisture and heat flow. The initial pore-water pressure distribution, $u_{wp0}$, is also required. Realistic functions for the amount of precipitation and evaporation are combined with the moisture and heat flow analysis to determine the transient soil-atmosphere boundary flux.

![Fig. 10. Factors affecting the stability of an embankment.](image-url)
In order to determine embankment stability, the pore-water pressure distribution is sampled at several pre-determined times, \( t = t_i \). The degree of saturation (or water content) within the soil mass and the soil unit weight distribution are computed as functions of the pore-water pressure distribution (i.e., the SWCC). For each of the chosen sampling times, the partial differential equations governing static equilibrium are solved using the soil unit weight distribution and external loads. Finally, the Dynamic Programming optimization algorithm uses the net total stresses and pore-water pressure distributions to determine the location and shape of the critical slip surface and the corresponding factor of safety.

The W-GHA model considers uncertainties related to soil properties and weather conditions through an appropriate frequency or probabilistic analysis. The probabilistic model consists of sampling the frequency distribution of the variables considered as uncertain. This is done a number of times and the deterministic model is run using each set of sampled values. The organization of several scenarios obtained from the discrete frequency distributions is presented in the format of a Decision Tree. The probability of the embankment becoming unstable is assessed based on the outcomes from the Decision Tree.

### 3.2 Partial differential equations governing soil behaviour

The hydro-thermo-mechanical behaviour of the soil comprising a railway embankment can be represented by a system of partial differential equations (PDE’s). These equations are obtained using a traditional continuum mechanics approach and appropriate state variables. The broadly accepted stress state variables; namely, net total stress (\( \sigma - u_\infty \)) and matric suction (\( u_{\text{sat}} - u_\infty \)), are used. The displacement state variables are the horizontal and vertical displacements, \( u \) and \( v \), and the change in volume of water and air in a referential volume. The PDE’s governing static force equilibrium, flow of moisture, and flow of heat are obtained from ba-
sic continuity and equilibrium laws, combined with constitutive laws that describe soil behaviour.

Moisture moves through soils driven by gradients of total head and/or partial pressures for each of the moisture phases (i.e., both liquid water and water vapour). The ratio between the flow of liquid water and water vapour depends mainly on the temperature and degree of saturation of the soil. Consequently, the transient temperature distribution needs to be taken into account when simulating the flow of moisture. Appropriate equations for the soil-atmosphere flux boundary conditions are required. The two-dimensional PDE’s used in this study are an extension of the one-dimensional formulations presented by Philip and de Vries (1957) and Wilson et al. (1994).

### 3.2.1 Conservation and flow of moisture

A series of assumptions form the backdrop for the equations accounting for the flow of liquid water and water vapour in soils. Some of the assumptions are as follows; namely, (i) the soil phases are individually continuous and therefore can be described using a continuum mechanics approach; (ii) the air phase is in permanent contact with the atmosphere; (iii) the flow of air due to thermal gradients has not been considered; (iv) osmotic pressure gradients are negligible; (v) local thermodynamic equilibrium between the liquid water and water vapour phases exists at all times at any point in the soil; (vi) temperature within the soil remains below the boiling point and above the freezing point of water at all times; (vii) dissolution of air into the liquid water phase is not considered; (viii) interphase water vapour flux caused by changes in partial vapour pressure and/or air phase volume is neglected; and (ix) hysteretic behaviour of the soil-water characteristic curve can be approximated by taking the average between the main drying and main wetting curves.

The conservation of mass equation for the water phase can be derived by taking the rate of flux of water mass in and out of a representative elemental volume (REV) and equating the difference to the rate of change of water mass within the element with time (Fig. 12). The x- and y-directions are considered for two-dimensional flow. Three types of water mass flow are considered; namely, mass flux of liquid water by advection ($\dot{q}_w^a$); mass flux of water vapour by diffusion ($\dot{q}_w^d$); mass flux of water vapour carried by bulk air by diffusion ($\dot{q}_w^v$). The mass flux of liquid water in unsaturated soils can be described by using Darcy’s law. The mass flux of water vapour can occur by diffusion and can be described using Fick’s law (Philip and de Vries 1957 and Dakshanamurthy and Fredlund 1981). Only the gradient in the partial water vapour pressure component of the total pressure gradient in air is considered (i.e., partial pressure of the dry air is considered constant and equal to the atmospheric pressure). The volume of water within the REV can be expressed in terms of the stress state variables. The coefficient of water volume change, $m_w^a$, can be written as the derivative of the soil-water characteristic curve. Combining the conservation of mass equation, the constitutive laws for water and air flow, and the derivative of the soil-water characteristic curve, the following PDE is obtained:
\[
\frac{\partial \left[ k^w \frac{\partial (u_w/\gamma_w)}{\partial x} + \frac{\bar{u}_a + p_v D^r}{\bar{u}_a} \frac{\partial p_w}{\partial x} \right]}{\partial x} + \frac{\partial \left[ k^w \frac{\partial (u_w/\gamma_w + y)}{\partial y} + \frac{\bar{u}_a + p_v D^r}{\bar{u}_a} \frac{\partial p_w}{\partial y} \right]}{\partial y} = -m_2^w \frac{\partial u_w}{\partial t}
\]

where \( k^w \) is the hydraulic conductivity \( (k^w = f(u_a - u_w), \text{m/s}) \); \( u_a \) is the pore-air pressure, kPa; \( u_w \) is the pore-water pressure, kPa; \( \gamma_w \) is the unit weight of water, \( \approx 9.81 \text{ kN/m}^3 \); \( y \) is the elevation, m; \( \bar{u}_a \) is the total pressure in the bulk air phase, \( u_{aim} + u_a \), kPa; \( p_v \) is the partial pressure of water vapour, kPa; \( D^r \) is the diffusion coefficient of the water vapour through the soil, \( (\text{kg.m})/(\text{kN.s}) \); \( \rho_w \) is the density of water, \( \approx 981.0 \text{ kg/m}^3 \); \( m_2^w \) is the coefficient of water volume change with respect to matric suction, \( m_2^w = (e/(1+e))(dS/d(u_a - u_w)) \), 1/kPa; and \( e \) is the void ratio.

A relationship between the partial pressure of water vapour and the total potential of the liquid pore-water (or total suction) can be obtained based on the thermodynamic theory of soil moisture (Edlefsen and Anderson 1943). Assuming local thermodynamic equilibrium, neglecting the effects of the osmotic suction, and assuming that the air pressure is equal to the atmospheric pressure, the following relationship applies:

\[
p_v = p_{vsat} \exp\left[ \frac{u_w g W_v}{\gamma_w R T} \right]
\]

where \( p_{vsat} \) is the saturation vapour pressure of the soil water at temperature \( T \), kPa; \( g \) is the acceleration of gravity, \( 9.81 \text{ m/s}^2 \); \( W_v \) is the molecular weight of water, \( 0.018016 \text{ kg/mol} \); \( R \) is the universal gas constant, \( 8.314 \text{ J/(mol.K)} \); and \( T \) is the temperature, K.

![Fig. 12. Soil representative elemental volume and water mass fluxes.](image-url)
According to Eq. 2, any of the three variables, $u_w, p_v$, and $T$, can be determined by the value of the other two variables. Therefore, the gradient of any of the three state variables is also determined by the gradient of the other two variables, as follows:

$$\nabla p_v = \frac{g W_v p_v}{\gamma_w R T} \left( \nabla u_w - \frac{u_w}{T} \nabla T \right) \quad (3)$$

Expressing the gradients of partial vapour pressure in Eq. 1 in terms of the gradients of pore-water pressure and temperature by using Eq. 3, the following PDE is obtained:

$$\frac{\partial}{\partial x} \left[ \left( \frac{k_w}{\gamma_w} + D^\omega \right) \frac{\partial u_w}{\partial x} - D^\omega \frac{u_w}{T} \frac{\partial T}{\partial x} \right]$$

$$+ \frac{\partial}{\partial y} \left[ \left( \frac{k_w}{\gamma_w} + D^\nu \right) \frac{\partial u_w}{\partial y} + k_w - D^\nu \frac{u_w}{T} \frac{\partial T}{\partial y} \right] = -m_2 \frac{\partial u_w}{\partial t} \quad (4)$$

where $D^\nu = \left( \overline{u}_a + \frac{p_v}{(\rho_w \overline{u}_a)} \right) \left( \overline{W}_v / (\rho_w R T) \right)$. Equation 4 is the final PDE governing the flow of moisture by liquid water and water vapour flow. Temperature gradients required to render this equation solvable can be obtained by solving the PDE satisfying conservation of thermal energy. Three unsaturated soil property functions can be identified in Eq. 4; namely: the hydraulic conductivity, the vapour diffusion coefficient, and the coefficient of water volume change. These soil properties functions vary with soil suction, and therefore the PDE is nonlinear.

### 3.2.2 Conservation and flow of heat

Heat transfer in soils occurs by three mechanisms, namely: conduction; convection; and latent heat due to phase change. Heat transfer by convection of the pore-fluid in soils is considerably smaller than conductive heat transfer (Milly 1984) and therefore can be neglected for the problem at hand. Vaporization is the only phase change presently of concern. Considering these conditions, the conductive and latent heat transfer in soils can be modelled using the Fourier equation (Dakshanamurthy and Fredlund 1981), as follows:

$$L_v \frac{\overline{u}_a + p_v}{\overline{u}_a} \frac{\partial}{\partial x} \left( D^\nu \frac{\partial p_v}{\partial x} \right) + L_v \frac{\overline{u}_a + p_v}{\overline{u}_a} \frac{\partial}{\partial y} \left( D^\nu \frac{\partial p_v}{\partial y} \right)$$

$$+ \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda \frac{\partial T}{\partial y} \right) = \zeta \frac{\partial T}{\partial t} \quad (5)$$

where $L_v$ is the latent heat of vaporization, $4.187 \times 10^3 (591 \ 0.51T)$, J/kg; $T$ is the temperature, °C; $\lambda$ is the thermal conductivity, $\lambda = f(u_a-u_w)$, W/(m °C); and $\zeta$ is the volumetric specific heat of soil, $\zeta = f(u_a-u_w)$, J/(m³ °C).
The first two terms in the left-hand side of Eq. 5 correspond to the heat energy released when there is a transfer of water between the vapour and liquid water phases (i.e., latent heat transfer by vaporization). Expressing the gradients of partial vapour pressure in Eq. 5 in terms of gradients of pore-water pressure and temperature by using Eq. 3, the following PDE is obtained:

\[
\frac{\partial}{\partial x} \left[ L_v D^* \rho_w \frac{\partial u_w}{\partial x} + \left( \frac{\lambda - L_v D^* \rho_w u_w}{T} \right) \frac{\partial T}{\partial x} \right] \\
+ \frac{\partial}{\partial y} \left[ L_v D^* \rho_w \frac{\partial u_w}{\partial y} + \left( \frac{\lambda - L_v D^* \rho_w u_w}{T} \right) \frac{\partial T}{\partial y} \right] = c_e \frac{\partial T}{\partial t}
\] (6)

The PDE for the flow of heat shown in Eq. 6 must be solved in a coupled manner along with Eq. 4. The primary variables are \( u_w \) and \( T \). Two unsaturated soil property functions can be identified in Eq. 6; namely: the thermal conductivity function and the volumetric specific heat. These soil properties functions also vary with soil suction, rendering the PDE non-linear. All of the above-mentioned unsaturated soil property functions bear a relationship to the SWCC.

### 3.2.3 Static equilibrium of forces and stress-strain relationship

The PDE's governing the static equilibrium of forces can be obtained by considering the equilibrium of forces acting upon a REV of soil. For the two-dimensional case, equilibrium in the \( x \)- and \( y \)-directions must be considered. The forces are expressed in terms of stresses and infinitesimal areas. Combining the equilibrium equations with Hooke's generalised stress-strain law, and expressing the strain in terms of small displacements, the following PDE's are obtained for the \( x \)- and \( y \)-directions:

\[
\frac{\partial}{\partial x} \left[ D_{11} \frac{\partial u}{\partial x} + D_{12} \frac{\partial v}{\partial y} \right] + \frac{\partial}{\partial y} \left[ D_{44} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] = 0
\] (7)

\[
\frac{\partial}{\partial x} \left[ D_{44} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] + \frac{\partial}{\partial y} \left[ D_{12} \frac{\partial u}{\partial x} + D_{11} \frac{\partial v}{\partial y} \right] + \gamma = 0
\] (8)

where \( u \) and \( v \) are the displacement in the \( x \)- and \( y \)-directions respectively; \( m \); \( D_{11} = E(1-\mu)/[(1+\mu)(1-2\mu)] \); \( D_{12} = E\mu/[(1+\mu)(1-2\mu)] \); \( D_{44} = E/[2(1+\mu)] \); \( E \) is the Young modulus, kPa; \( \mu \) is the Poisson ratio; \( \gamma = (G_s + S_e) y_w / (1 + e) \) is the unit weight of the soil, kN/m³; and \( G_s \) is the specific unit weight of soil particles.

Overall volume changes in the soil due to changes in pore-water pressure are neglected in Eqs. 7 and 8. If the volume changes due to changes in pore-water pressure are to be considered, constitutive relationships between volume change and pore-water pressure would have to be considered and Eqs. 7 and 8 would have to be solved in a coupled manner with Eqs. 4 and 6. The primary variable of interest in the W-GHA model is the change in pore-water pressure and net total
stresses in response to the atmospheric boundary conditions. Therefore, the stress
equilibrium-moisture flow coupling would not appear to be essential.

The system formed by Eqs. 4, 6, 7, and 8 represents a thermal-hygro-
mechanical model appropriate for the application at hand. In order to solve these
equations, the W-GHA model has made use of a multi-purpose partial differential
equation solver, FlexPDE (PDE solutions 2003). FlexPDE uses the Finite Ele-
ment (FE) and the Finite Difference (FD) methods combined with Newton-type
methods of solution of non-linear coupled systems. The friendly input and output
features combined with automatic mesh generation, time-step control, and choice
of non-linear approaches makes FlexPDE a powerful part of a complete problem
is another example of a PSE developed specifically for geotechnical engineering
problems.

3.3 Weather-related boundary conditions

The partial differential equations based on the conservation of moisture and heat
require boundary conditions related to atmospheric forcing conditions. The net
soil-atmosphere moisture flux is a function of some of the key components of the
hydrological cycle; namely, precipitation, actual evaporation, and run-off. Other
components, such as depression storage, interception, and plant transpiration, are
not considered herein, but could also be included in the formulation. The heat
flow at the soil-atmosphere boundary is a function of the net radiation available at
ground surface and the latent heat of evaporation. Appropriate equations to repre-
sent these boundary conditions are presented in the following sections.

3.3.1 Net soil-atmosphere moisture flux

The combination of precipitation, actual evaporation, and runoff can produce a net
soil-atmosphere boundary flux designated either as infiltration (positive flux) or
exfiltration (negative flux). The amount of net soil-atmosphere moisture flux must
satisfy the following water balance equation:

\[ NF = P - AE - R \]  

(9)

where \( NF \) is the net moisture flux, m/s; \( P \) is the precipitation, m/s; \( AE \) is the actual
evaporation, m/s; and \( R \) is the runoff, m/s. The net moisture flux must be applied
as the component of the vertical vector of magnitude \( NF \), normal to the ground
surface and as a natural boundary condition to Eq. 4. The amount of precipitation,
\( P \), will be a known input based upon weather data. The amount of actual evapora-
tion, \( AE \), and runoff, \( R \), are a function of both weather and soil conditions. More
specifically, \( AE \) and \( R \) are a function of the soil suction at the soil-atmosphere
boundary. As a result, the net moisture flux is an unknown that must be computed
by simultaneously solving the net soil-atmosphere moisture flux boundary condi-
tion and the PDE's governing the movement of water and heat.
The amount of actual evaporation is a function of potential evaporation, \( PE \) (i.e., the amount of evaporation from a water surface under specific atmospheric conditions), and the soil surface conditions. The flow of moisture towards the soil surface for wet conditions occurs primarily as liquid water movement. As the soil dries, water begins to move in the form of vapour. A decrease in water content at the soil surface corresponds to an increase in soil suction. As soil suction increases, a larger amount of energy is required to remove water from the soil surface. Wilson et al. (1997) demonstrated that the actual evaporation from a soil surface can be determined by using a measure of potential evaporation combined with a limiting function. This limiting function reflects the decrease in actual evaporation as soil suction at the boundary is increased:

\[
AE = PE \left[ \frac{RH - \left( \frac{P_{\text{vast}}}{P_{\text{vast}}} \right) RH_{\text{air}}}{1 - \left( \frac{P_{\text{vast}}}{P_{\text{vast}}} \right) RH_{\text{air}}} \right]
\]  

where \( RH = P_r / P_{\text{vast}} \) is the relative humidity at the soil surface, given by Eq. 2; \( P_{\text{vast}} \) is the saturation vapour pressure of the air, kPa; and \( RH_{\text{air}} \) is the relative humidity of the air near the ground surface. According to Eq. 2, as soil suction increases, \( RH \) decreases until it eventually approaches zero for a value of suction approximately equal to \( 1 \times 10^5 \) kPa. Likewise, Eq. 10 shows that as the relative humidity decreases, \( AE \) decreases until it approaches zero when the relative humidity approaches zero. If a direct measure of potential evaporation is not available, it is possible to use one of the several equations proposed in the literature to calculate \( PE \) based on weather data.

The net moisture flux at the soil-atmosphere boundary (Eq. 9) can be determined once the amount of precipitation is known and the parameters of the \( AE \) equation are obtained. The third component in Eq. 9 is runoff and it can be computed in an interactive way. If the embankment being analysed has an effective drainage system, any runoff water will be removed from the ground surface. In this case, the amount of net moisture flux, \( NF \), should not produce pore-water pressures at ground surface higher than zero. Therefore, the following set of conditions is used:

\[
NF = \begin{cases} 
  P - AE & \text{if } P - AE > 0 \text{ and } u_{\text{ws}} < 0 \\
  EF(0 - u_{\text{ws}}) & \text{if } P - AE > 0 \text{ and } u_{\text{ws}} \geq 0 \\
  P - AE & \text{if } P - AE \leq 0 
\end{cases}
\]

where \( u_{\text{ws}} \) is the pore-water pressure at the surface, kPa; and \( EF \) is simply a large number, such as \( 1 \times 10^6 \). If the multiplier \( EF \) tends to infinity, the area flux boundary condition \( NF = EF(0 - u_{\text{ws}}) \) becomes mathematically equivalent to the boundary node value pore-water pressure, \( u_{\text{ws}} = 0 \). Therefore, the boundary condition using the flux [\( EF(0 - u_{\text{ws}}) \)] is used as an alternative to switching the category of boundary condition when the pore-water pressure at the soil surface reaches zero. Equation 11 has been implemented in FlexPDE using conditional functions built into the software. Runoff can occur when the potential infiltration is sufficiently
large and the hydraulic conductivity is small relative to the water available for infiltration. The amount of runoff corresponds to the difference between the water available (i.e., \( P - AE \)), minus the amount of net flux computed in the interactive manner from Eq. 11.

3.3.2 Heat flow

The heat flow at the ground surface must be in accordance with the following energy balance equation:

\[
H = Q_n - AE
\]

where \( H \) is the heat flux at the soil surface, \( \text{W/m}^2 \); \( Q_n \) is the net radiation available at the soil surface, \( \text{W/m}^2 \); and \( AE \) is the actual evaporation, \( \text{W/m}^2 \). The heat flux \( H \) must be applied as a natural (flux) boundary condition, to Eq. 6.

3.4 Stability analysis using Dynamic Programming

The Dynamic Programming Method (DPM) is a general method of maximization and minimization of linear (additive) functionals (Bellman 1957). Baker (1980) applied the DPM to slope stability problems, while retaining the Spencer (1967) assumption regarding interslice forces. Yamagami and Ueta (1988) extended the DPM approach and replaced the Spencer limit equilibrium method with stresses computed from a Finite Element stress analysis. The result of DPM studies have also been presented by Zou et al. (1995), Pham et al. (2001), and Pham (2002).

Conventional limit equilibrium methods are usually combined with an assumption regarding the shape of the critical slip surface. The DPM represents an important breakthrough in that the slip surface shape restrictions are significantly relaxed. The combined use of Finite Element stress fields further enhances the analysis by providing a means of incorporating into the slope stability analysis more realistic boundary conditions, soil stress-strain properties, and stress history. Little human interference in the analysis process is required once the problem geometry, boundary conditions, material properties, and search grid are established.

The input requirements for the DPM combined with Finite Element stress fields are quite similar to those of conventional limit equilibrium methods. The stress-strain constitutive parameters and the boundary conditions are the only additional parameters that must be designated. The use of a problem solving environment, such as FlexPDE, can make stress-strain analyses as easy as traditional limit equilibrium procedures since only approximate values of the elasticity parameters are required (Pham 2002). If a linear constitutive law is adopted in the stress analysis, the computational time can be similar to, or shorter than that of conventional limit equilibrium codes (Pham et al. 2001). The characteristics of the DPM combined with Finite Element stress fields are particularly desirable for the W-GHA model because all analyses are reduced to the solution of a series of partial differential equations.
### 3.4.1 Optimization procedure

Figure 13 presents the analytical scheme for stability analysis using the Dynamic Programming Method, DPM (Yamagami and Ueta 1988 and Pham et al. 2001). The procedure used in the W-GHA model is based on the assumption that the critical slip surface can be defined using \( n \) linear segments. Each linear segment connects two state points located at two successive stages. The search grid consists of a collection of state points lying on \( n+1 \) stages. A relatively coarse search grid is superimposed on the geometry shown in Figure 13. The dynamic programming procedure is used to determine a continuous assemblage of segments that corresponds to a minimum overall factor of safety. In order to quantify the overall stability, a factor of safety, \( F_s \), along a slip surface is defined in its discrete form as:

\[
F_s = \frac{\sum_{i=1}^{n} R_i}{\sum_{i=1}^{n} S_i} = \frac{\sum_{i=1}^{n} \tau_f \Delta L_i}{\sum_{i=1}^{n} \tau_i \Delta L_i} \tag{13}
\]

where \( n \) is the total number of segments; \( R_i \) is the resisting force of the soil along the \( i^{th} \) segment, kN/m; \( S_i \) is the shear force acting along the \( i^{th} \) segment, kN/m; \( \tau_f \) is the shear strength of the soil along the \( i^{th} \) segment, kPa; \( \Delta L_i \) is the length of the \( i^{th} \) segment, m; and \( \tau_i \) is the shear stress along the \( i^{th} \) segment, kPa. This definition of factor of safety (i.e., considering the state of overall limit equilibrium of forces), puts the DPM within a similar class to the limit equilibrium methods. The unique characteristic of the DPM lies in the manner by which the shape and position of the critical slip surface, along with the corresponding minimum \( F_s \), is obtained.

In order to minimize the non-additive \( F_s \) functional the following additive functional is introduced (Baker, 1980):

![Fig. 13. Analytical scheme for stability analysis using Dynamic Programming Method.](image)
\[ G = \sum_{i=1}^{n} (\tau_j \Delta L_i - F_s \tau_i \Delta L) \]  

(14)

In order to minimise \( G \), the optimal function, \( H \), is used. \( H_{i+1}(j) \) is defined as the minimum of \( G \) between any state point at the initial stage and any posterior state point \([i, j]\). According to the principle of optimality (Bellman 1957), the optimal function at a posterior stage, \( H_{i+1}(j) \), is a function of the optimal function at the prior stage, \( H_i(k) \), as follows:

\[ H_{i+1}(j) = \min[H_i(k) + DG_{i+1}(j, k)] \]  

(15)

where \( i = 1, n; j = 1, NP(i+1); k = 1, NP(i); NP(i) \) is the number of state points on stage \( i \); and \( DG_{i+1}(j, k) = \tau_j \Delta L_i - F_s \tau_i \Delta L \). The term \( DG_{i+1}(j, k) \) corresponds to the 'cost' of passing between the state points \([i+1, j]\) and \([i, k]\), and is termed the return function. The principle of optimality establishes that the optimal function at each state point of a posterior stage corresponds to the minimum value amongst all the sums of the optimal functions at each state point of the prior stage and the 'costs' of passing between the state points \([i+1, j]\) and \([i, k]\). The optimal path that defines the critical slip surface is found by connecting the optimal state points, traced back from the final stage to the initial stage. The value of \( F_s \) is given an assumed value for the first slip surface computation and replaced by the newly obtained \( F_s \). This process must be repeated until the \( F_s \) converges to a unique value while respecting an error tolerance.

### 3.4.2 Shear strength and stresses within the DPM search grid

The shear strength, normal stress, and acting shear stress at all state points must be determined in order to solve Eqs. 14 and 15. These variables can be calculated based on the finite element stress field, pore-water pressure field, shear strength parameters, and orientation angle of each slip surface segment. The stress field is obtained by solving the PDE's governing static equilibrium (i.e., Eqs. 7 and 8). The pore-water pressure field is determined by solving the PDE's governing water (and heat) movement along with appropriate soil-atmospheric boundary conditions (Eqs. 4, 6, and 12).

The shear strength, \( \tau_f \), can be determined by using the shear strength envelope for a saturated/unsaturated soil (Fredlund and Rahardjo 1993). The shear strength with respect to soil suction can be based on a prediction technique dependent upon the SWCC. The normal stress, \( \sigma_n \), and shear stress, \( \tau \), acting on a plane inclined at an angle \( \theta \) can be computed from the stress state defined by \( \sigma_x \), \( \sigma_y \), and \( \tau_{xy} \):

\[ \sigma_n = \sigma_x \sin^2 \theta + \sigma_y \cos^2 \theta - \tau_{xy} \sin 2\theta \]  

(16)

\[ \tau = [(\sigma_x - \sigma_y)/2] \sin 2\theta - \tau_{xy} \cos 2\theta \]  

(17)

where \( \sigma_x \) and \( \sigma_y \) are the total normal stresses acting on the \( x \)- and \( y \)-directions, respectively; and \( \tau_{xy} \) is the total shear stress acting on the \( x \)-plane and \( y \)-direction.
The normal stress, $\sigma_n$, (Eq. 16) is later used in the calculation of shear strength. The acting shear stress, $\tau_n$ (Eq. 17) is required in the factor of safety equation. A computer model called SAFE-DP (Stability Analysis using Finite Element stress fields and Dynamic Programming) was written and integrated with Flex-PDE. SAFE-DP performs the DPM analysis based on a finite element stress analysis, along with the pore-water pressure fields. The model can accommodate complex geometries and stratigraphy.

4 UNSATURATED SOIL PROPERTIES ASSESSMENT

Table 1 presents a summary of the soil properties required by the W-GHA model. The soil properties are found in the PDE's governing the hydro-thermo-mechanical behaviour of the soil comprising a railway embankment (Eqs. 4, 6, 7, and 8). The shear strength properties required by the Dynamic Programming optimization are also listed. A large number of soil properties are required in order to access weather-related embankment hazards. The properties associated with unsaturated soil behaviour are particularly difficult to measure. Figure 14 presents the approaches that can be taken for the assessment of unsaturated soil properties. A variety of direct (laboratory and field) and indirect (prediction) methods can be used. Laboratory and field approaches are usually complex, costly, and time consuming. It is not feasible to implement the W-GHA model if demanding test procedures are required for the assessment of unsaturated soil properties.

The second main branch of Fig. 14 presents two estimation approaches that can be used. These approximate methods rely on soil data that is simpler, and easier to obtain. The use of estimation methods results in additional uncertainty to the embankment stability measure. However, the uncertainties can be rationally assessed and taken into account in the analyses using probabilistic approaches.

<table>
<thead>
<tr>
<th>Property</th>
<th>Parameters</th>
<th>(*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moisture flow</td>
<td>$m_w^2$, $\psi_b$, $\psi_{res}$, $S_{res}$, $a$, $m_v$, $e$.</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>$k^w$, $\psi_b$, $\psi_{res}$, $S_{res}$, $a$, $k_{sat}$, $\eta$.</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>$D^w$, $\psi_b$, $\psi_{res}$, $S_{res}$, $a$, $e$, $D_{mp}$.</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>$L_f$</td>
<td>1</td>
</tr>
<tr>
<td>Heat flow</td>
<td>$\lambda$, $\psi_b$, $\psi_{res}$, $S_{res}$, $a$, $\lambda_3$, $\lambda_{w1}$, $\lambda_{w2}$.</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>$\zeta$, $\psi_b$, $\psi_{res}$, $S_{res}$, $a$, $e$, $C_s$, $C_w$.</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>$D_{ij}$</td>
<td>$E$, $\mu$.</td>
</tr>
<tr>
<td>Stress and stability</td>
<td>$\gamma_y$, $\psi_b$, $\psi_{res}$, $S_{res}$, $a$, $e$, $G_s$.</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>$\tau_f$, $\psi_b$, $\psi_{res}$, $S_{res}$, $a$, $c'$, $\phi'$, $\kappa$.</td>
<td>3</td>
</tr>
</tbody>
</table>

(*) - Number of exclusive properties.
Fig. 14. Approaches to determine the unsaturated soil property functions.

Unsaturated soil properties are primarily a function of the amount of water stored in the soil, amongst other secondary factors. Therefore, the estimation of unsaturated soil properties relies on the physical significance of the soil-water characteristic curve. Most unsaturated property functions can be shown to be a function of the saturated soil properties and the soil-water characteristic curve, as shown in Figure 15. Thermal properties could also be listed in Fig 15 as being dependent on the SWCC. The next sections present some estimation techniques for obtaining unsaturated soil property functions using the SWCC. A method of prediction of the SWCC based on the grain-size distribution can be found in Fredlund (1999).

An estimation of an appropriate SWCC for any particular soil can be obtained from a database such as that available through SoilVision (1996). Either a description of the soil or a grain-size distribution curve can be used to proceed with 'mining' or 'querying' the database for appropriate SWCC information. The SoilVision computer software also can be used to compute all the corresponding unsaturated soil property functions.

4.1 Hydraulic conductivity

The methods of assessing the hydraulic conductivity function can be classified as empirical equations, mechanistic models, and statistical models. Huang et al. (1998) presents a summary of the available methods. There is a clear relationship between the SWCC and the mechanistic and statistical models. Mechanistic models are based on the application of the capillary theory to the soil pores. The soil-water characteristic curve is used to indicate the size of the water-filled pores. Brooks and Corey (1964) developed the following equation using the mechanistic approach:

\[
k = k_{sat}^\omega \text{ for } \psi < \psi_b
\]

\[
k = k_{sat}^\omega \left[\frac{\psi_b}{\psi}\right]^n \text{ for } \psi \geq \psi_b
\]

(18)
where $k_{s,sw}^w$ is the saturated hydraulic conductivity, $\eta = 2 + 3\lambda$, $\lambda = \log S_e^/\log \psi$, and $S_e = [(S - S_{res})/(1 - S_{res})]$. The portion of the SWCC past the air entry value is assumed to follow a straight line when plotted in the diagram log $S_e$ vs. log $\psi$. The mechanistic equations have a theoretical basis and have been repeatedly tested against experimental data (Mualem 1976). The mechanistic equations are also easy to use.

Statistical models are based on a random variation of pore sizes. The SWCC is again used to access the size of the pores based on the soil suction. Childs and Collis-George (1950) present one of the well known methods. Fredlund et al. (1994) proposed a permeability function that uses a similar approach, but applies the Fredlund and Xing (1994) SWCC equation. There are limitations associated with applying these equations to clayey soils.

### 4.2 Diffusion coefficient of vapour through soil

The diffusion coefficient of water vapour through soil, $D^v$, can be predicted using the following equation:
\[ D' = \alpha \beta D_{\text{vap}} W_v / RT \]  
where \( \alpha \) is the tortuosity factor of the soil, \( \beta = \beta^{2/3} \), \( \beta \) is the cross-sectional area of soil available for vapour flow per total area, \( \beta = (1 - S)n \); \( S = V_w/V \); \( n \) is the soil porosity; \( D_{\text{vap}} \) is the molecular diffusivity of water vapour in air, \( D_{\text{vap}} = 0.229 \times 10^{-4} (1 + T / 273.15)^{1.75} \text{ m}^2/\text{s} \); \( W_v \) is the molecular weight of water, 18.016 kg/kmol; \( R \) is the universal gas constant, 8.314 J/(mol.K); \( n \) is the temperature, K. Equation 19 shows that \( D' \) is a function of \( S \) and \( n \), which in turn are functions of soil suction (i.e., the SWCC).

4.3 Thermal properties

The volumetric heat capacity and the thermal conductivity of the soil can be calculated by proportioning air, water, and soils by volume and using the thermal properties of each phase (de Vries 1963). The equation for the heat capacity of the soil, neglecting the heat capacity of the air phase, is as follows:

\[ \zeta = \zeta_s \frac{V_s}{V} + \zeta_w \frac{V_w}{V} = \zeta_s (1 - n) + \zeta_w n S \]  

where \( \zeta_s \) is the volumetric specific heat of solids, 2.235x10^6 J/(m³.°C); and \( \zeta_w \) is the volumetric specific heat of water, 4.154x10^6 at 35°C, J/(m³.°C).

The equation for the assessment of the thermal conductivity function proposed by de Vries (1963) is as follows:

\[ \lambda = \left( F_s \lambda_s \frac{V_s}{V} + F_w \lambda_w \frac{V_w}{V} + F_a \lambda_a \frac{V_a}{V} \right) \left( F_s \frac{V_s}{V} + F_w \frac{V_w}{V} + F_a \frac{V_a}{V} \right) \]  

where \( \lambda_s \) is the thermal conductivity of solids, typically around \( \lambda_s = 6.0 \), W/(m °C); \( \lambda_w \) is the thermal conductivity of water, typically around \( \lambda_w = 0.57 \), W/(m °C); \( \lambda_a = \lambda_{da} + \lambda_{va} \); \( \lambda_{da} \) is the thermal conductivity of dry air, typically around \( \lambda_{da} = 0.025 \), W/(m °C); \( \lambda_{va} \) is the thermal conductivity of water vapour, assumed as \( \lambda_{va} = (0.0736)S \), W/(m °C); \( F_{a,s} = 1/3 \sum_{i=1}^3 \left[ 1 + \left( \lambda_{a,s} / \lambda_{a,v} - 1 \right) g_i \right]^{-1} \); \( F_w = 1.0 \) (water assumed as the continuum medium); \( g_{1,2} = 0.015 + (0.333 - 0.015)S \) (assuming spherical particles); and \( g_3 = 1 - g_1 - g_2 \).

4.4 Shear strength

The shear strength envelope for an unsaturated soil can be predicted using the soil-water characteristic curve and the saturated shear strength parameters, \( c' \) and \( \phi' \). Theoretical models supported by experimental evidence show that the slope of the plot of shear strength versus soil suction, \( \phi' \), begins to deviate from the effective angle of internal friction as the soil desaturates. The reduced slope is associated...
with a reduction in the effective wetted area of contact past the air-entry value (Fredlund et al. 1996 and Vanapalli et al. 1996):

\[ \tau_{f_i} = c' + (\sigma_{n_i} - u_a) \tan \phi' + (u_a - u_w) \Theta \tan \phi' \]

(22)

where \( \Theta = S \); and \( \kappa \) is a fitting parameter to account for any non-linearity between the area and volume representation of the amount of water contributing to the shear strength. Vanapalli et al. (1996) presents a second procedure, defining \( \Theta = \Theta_e = (S - S_{res})/(1 - S_{res}) \) and not requiring \( \kappa \). This second procedure renders the envelope potentially less flexible once the fitting parameter \( \kappa \) is not used. However, the use of \( \Theta_e \) may be interpreted as a direct method for accounting for the same non-linearity that \( \kappa \) accounts for. The use of \( \Theta \) set equal to \( S \) seems appropriate for the W-GHA model since shear strength at failure can be applied along the entire soil suction range.

5 FREQUENCY AND SENSITIVITY ANALYSIS

The assessment of weather-related geo-hazards is done in terms of the frequency distribution of a measure of stability (i.e., the Factor of Safety). The choice of variables that must be modelled as uncertainties (i.e., probabilistically modelled) is based on sensitivity analyses. The Decision Analysis framework adopted by the W-GHA model provides an efficient and systematic environment for frequency and sensitivity analyses.

5.1 Frequency (probabilistic) analysis

Frequency analyses correspond to step 4 of the Decision Analysis cycle (see Fig. 7). In order to perform frequency analyses, the relationships, dependences (correlations), and frequency distributions of the involved parameters must be established. The relationships between variables correspond to the previously presented model of the problem. These relationships can be illustrated using an Influence Diagram for the W-GHA model (Fig. 16). The variables presented inside the round boxes are the uncertain variables. The variables presented inside the square boxes are certain (fixed) values. These values are included for clarity and informative purposes, and could be included in the deterministic equations without being explicitly shown on the Influence Diagram. The arrows indicate the relationships between the parameters and variables. The Factor of Safety box is the final outcome, or the direct and indirect recipient of all arrows. The decision as to which variables must be modelled as uncertainties and which correlations must be taken into account depends on the sensitivity of each variable in the computation of the stability of the railway embankment. The uncertainties indicated in Fig. 16 are preliminary and form an illustrative Influence Diagram for the W-GHA model since the study is still underway.
A discrete stochastic analysis has been applied to the embankment hazard problem. According to this efficient approach, the frequency distributions of the uncertain variables can be represented by a few sampling points. The reduced numbers of sampling points produces lower computational effort while not significantly compromising the results (Clemen 1996). Discrete frequency distributions must be represented using complementary scenarios (i.e., scenarios whose sum of probabilities is “1”). For example, the air-entry value of the soil comprising a railway embankment may be modelled as an uncertain variable using a normal distribution with a determined mean and standard deviation. The entire continuous distribution can be presented using three scenarios; namely, a low, a mean (or nominal), and a high air-entry value. The value and probability of each scenario is assigned based on the given standard deviation, while adding up to “1”.

The presence of several uncertain variables produces a combination of different scenarios. The presentation of several scenarios is called the Decision Tree. Figure 17 presents the Decision Tree corresponding to the Influence Diagram shown in Fig. 16. Three branches are used for each variable. The notation used here to present the Decision Tree is from the popular Decision Programming language called DPL (Applied Decision Analysis LLC 1998). In the notation of Fig. 17 the branches are suppressed, improving the clarity of the Decision Tree and allowing the presentation of more information in less space. Ultimately, 729 scenarios \(3^6\) can be obtained from the 6 uncertain variables represented by three branches.

Informative results have been obtained thus far in the study; however, some limitations of the model became apparent. Hazard assessment problems of extremely low failure frequency are difficult to solve. The solution becomes quite sensitive to the number points sampled and to the frequency distribution equations adopted. On the other hand, the accuracy of the computation of extremely low failure frequencies is less critical because this type of condition is usually within acceptable hazard levels for the problem at hand.

![Fig. 16. An influence diagram for the W-GHA model.](image-url)
5.2 Sensitivity analysis

The determination of those variables that are of reduced importance is paramount to the success of the W-GHA model. For example, if the residual degree of saturation in Fig. 16 is found to have a relatively lower sensitivity to the Factor of Safety, it could be modelled as a known, fixed value, reducing the number of scenarios to 343 (i.e., 3^5). Tools and techniques common to Decision Analysis have been used to determine the variables that are most sensitive within the analysis. Rainbow diagrams, expected value tornado diagrams, and event tornado diagrams have been constructed for some cases. A type of “Value of Information” analysis is also being used to allow researchers and CP Rail officials to focus investigations on the parameters that have the greatest effect on the stability of an embankment. Such information allows an optimized allocation of funds for investigation, research, and monitoring.

Preliminary analyses indicate that precipitation is the most sensitive variable in most regions, and therefore the assessment of precipitation distributions becomes a priority. However, there may be some regions with different climates where potential evaporation is as important as precipitation, and both must be assigned the same level of priority. The sensitivity analyses are being conducted using hypothetical cases. The cases embrace a range of typical cross-sections along the Canadian railway system.

6 FINAL REMARKS

A model for the assessment of weather-related geo-hazards (W-GHA model) has been presented. The W-GHA model consists of a slope stability analysis on a two-dimensional embankment combined with the analysis of the effects of weather conditions on the pore-water pressures. A discrete stochastic approach has been implemented within the proposed framework. Unsaturated soil mechanics has been taken from the soil property assessment level using techniques based on the soil-water characteristic curve (SWCC), to the assessment of weather-related geo-hazards.
A series of partial differential equations governing the thermo-hydro-mechanical behaviour of saturated/unsaturated soils has been presented, along with appropriate soil-atmosphere boundary conditions. Robust problem solving environments, such as FlexPDE, make the solution of the rigorous series of PDE's relatively simple and quick. A slope stability algorithm called SAFE-DP using the Dynamic Programming method has been developed and incorporated into the W-GHA model. Stability conditions can be computed using SAFE-DP, the stress distribution, and the pore-water pressure distribution that reflect the weather conditions.

The unsaturated soil property functions required as input to the W-GHA model have been presented along with a relatively simple prediction methodology. The prediction methods are based on the relationship between unsaturated soil behaviour and the SWCC. As a result, a reduced number of soil properties are required for the implementation of the W-GHA model into engineering practice. The paper illustrates the feasibility of quantitatively assessing embankment stability based on weather conditions.

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