Large-Strain 1D, 2D, and 3D Consolidation Modeling of Mine Tailings

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ABSTRACT: The management of the consolidation process in mine tailings is central to the long-term behavior of tailings management areas (TMA). The capacity of such TMAs as well as the long-term environmental impacts of such facilities all originate from the consolidation process which will occur over time. The typical slurried deposition of such tailings causes large potential consolidation of the tailings and therefore large-strain theory applies to this problem. The software tools for the application of large-strain theory have been slow in coming given that the mathematical theory was defined in the 60's and 70's. This paper examines the application of a 1D, 2D, and 3D software finite element tool to the solution of large-strain consolidation as applied to TMAs. Expected differences between small-strain and large-strain tools are outlined. Differences between a 1D, 2D, and 3D analysis will be examined and the question "Is a 1D analysis sufficient?" will be answered.

1 INTRODUCTION

The theory of small-strain consolidation theory was first defined by Terzaghi (1923, 1936) and is recognized to be based on a number of simplifying assumptions. The difficulty of the theory was i) developing methods of solution of the equations (prior to the computer) and ii) applying the theory to real-world problems. The Terzaghi solution assumes no relevant change in permeability as the soil deforms and therefore renders the governing partial differential equation easy to solve.

Early work by the phosphate industry in the 60's and 70's extended the consolidation formulations to deal with multiple deposition layers and large-strain consolidation. It was generally recognized that the large consolidation deformations when starting with initial void ratios as high as 15 became problematic when working with small-strain formulations. Formulations also allowed for the non-linear behavior of void ratio and therefore permeability which are necessary for the analysis of large-strain problems. A number of researchers in this time period developed 1D large-strain formulations in which the non-linear changes in permeability and void ratio are taken into account (Davis and Raymond, 1965; Schiffman, 1958; and Bardon and Berry, 1965). Additional theoretical formulations where the changes in self-weight are accounted for may be found in the equations developed by (Mikasa, 1965; Gibson et al, 1967, 1981; and Lee and Sills, 1979).

The difficulty of these early formulations is that many were formulated in terms of void ratio in 1D and therefore not consistent with general stress / deformation formulations.

SoilVision Systems Ltd. has undertaken a research effort in the past few years in order to extend the large-strain formulations to 2D and 3D in a manner consistent with traditional stress-deformation formulations based on stress-states rather than void ratios.

The developed formulations are ideal for the prediction of long-term consolidation of mine tailings. Their application to the estimation of long-term tailings behavior through use of a 2D and 3D large-strain consolidation software tool is a relatively new application field. It is therefore the intent of this paper to i) demonstrate reasonable consistency with existing benchmarks and ii) answer basic questions related to the application of such a tool to tailings management.

2 BENCHMARKING / VERIFICATION

Benchmarking of software implementing new theory is a crucial yet difficult part of any new software package. Benchmarking is a fundamental part of QAQC which ensures that a computer code can correctly reproduce known solutions to problems generated using both analytical and numerical methods. The theory was added to an internal version of the SVFLUX / SVSOLID coupled software packages as developed in the context of the SVOFFICE 2009 geotechnical software office suite. Each package has been extensively benchmarked in the areas of flow and stress deformation as viewable in their respective verification manuals. Therefore the benchmarking effort focused on the new areas of implementation; specifically i) large-strain deformation and coupled consolidation analysis.

A series of benchmarks are presented in the following sections which illustrate benchmarking of the various components of the large-strain consolidation solution. The benchmarks selected consisted of the following:

i) Large-strain uncoupled theory
ii) Coupling of small-strain / large-strain equations
iii) 1D Coupled Large-Strain Benchmark

The solution of the benchmarks form the basis for reasonable confidence that the coupled software is i) performing reasonably and ii) is consistent with previously documented work.

2.1 *Small-strain / Large – strain Uncoupled*

Of foremost and fundamental importance is determining that the uncoupled large-strain theory is being properly solved. Large-strain theory involves the solution of the stress / deformation equations using a lagrangian reference frame. This inherently means that the mesh nodal points move with the deformations.

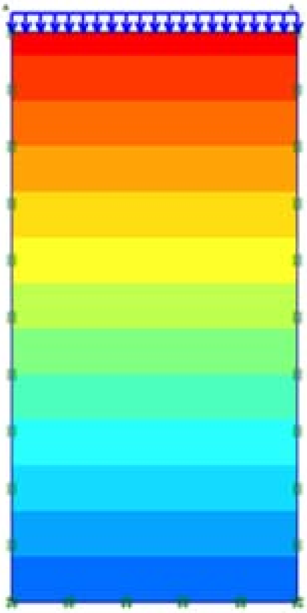

Figure 1 Example 1D geometry used for large-strain verification

In order to test the correct implementation of large-strain analysis a simple 1m high column model was created in the FLAC software, in a mixed Eularian-Lagrangian software, and in the SVSOLID software. An example of the 1D column may be seen in Figure 1. A sufficient load was applied to the column such that large-strain deformations were initiated (> 10%).

After the model was run it was found that a small-strain model produced a deformation = 0.5m. FLAC produced a deformation of 0.4m and the Mixed Eularian-Lagrangian software produced a deformation of 0.33m. SVSOLID could be set to duplicate any of the small-strain or large-strain solutions simply by adjusting aspects of its formulation. The SVSOLID software matched the answers of the other software packages exactly.

It is also important to note that a small-strain solution to a large-strain model will tend to overestimate the deformations. This is demonstrated in this example model in which the small-strain model estimates the highest amount of deformations (0.5m).

2.2 *Coupled Benchmarking – Mandel / Cryer*

An important and difficult aspect of a consolidation solution to document is the coupling mechanism. For this part of the software the Mandel-Cryer benchmark was chosen. In this benchmark a sphere with zero pore-water pressure is abruptly loaded. The load is transferred to the pore-water. The areas near the sphere boundary are allowed to drain faster than the regions closer to the center. The consolidation of the outer layers causes shrinkage of a thin cap, which transmits extra loading to the sphere core. As a result, pore-water pressures at the center of the sphere rise during a period of time higher than the external load. This effect can only be simulated by solving the fully coupled consolidation equations. The effect can graphically be seen in Figure 2.

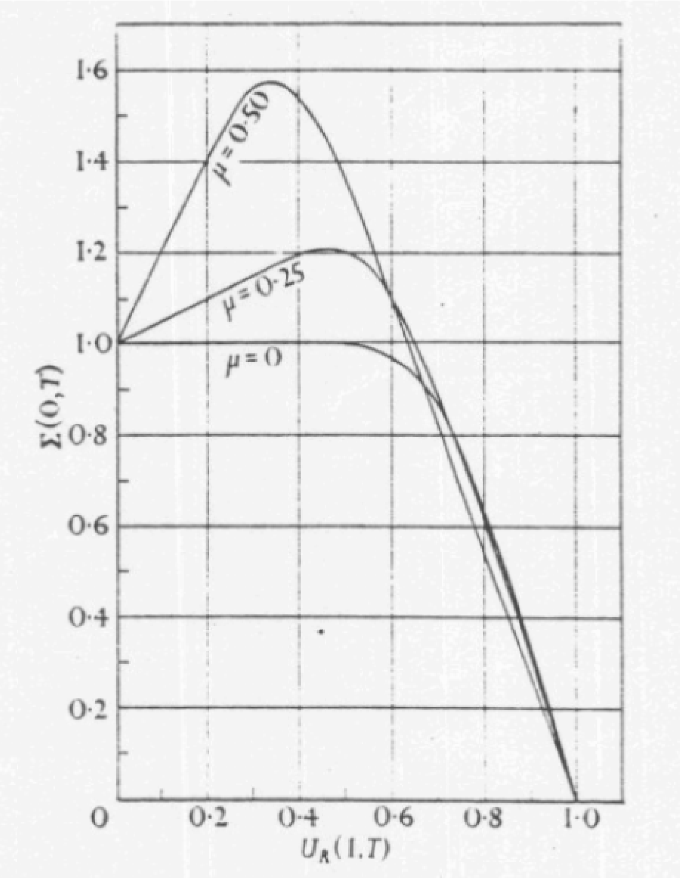

Figure 2 Normalized strain (y-axis) versus normalized time (x-axis) may be seen in Cryer's (1963) figure

It should also be noted that the amount of rise in pore-water pressures changed based on the Poisson's Ratio used for the analysis.

The coupled SVSOLID / SVFLUX numerical model was able to demonstrate reasonable comparison with the published answers as seen in Figure 3. The answers differ primarily in presentation as a different normalization technique for time was used in Cryer's original paper (a logarithmic technique). Cryer's original paper also does not present loading times where the SVFLUX / SVSOLID results present loading times. Overall the comparison is reasonable and demonstrates that the coupling in SVSOLID / SVFLUX is implemented properly.

Gibson (1990) review the potential impact that the Mandel-Cryer would have on large-strain formulations with variable permeability. The effect was known to raise pore-water pressures to approximately 60% higher than applied loads in a small-strain problem.

The study by Gibson found that the differential decrease in permeability during consolidation is of dominant importance. Both the magnitude and the rate of dissipation of pore pressure diminish with increasing stress. This is likely due to the throttling effect of a sudden drop in the skeleton permeability close to the drainage surface.

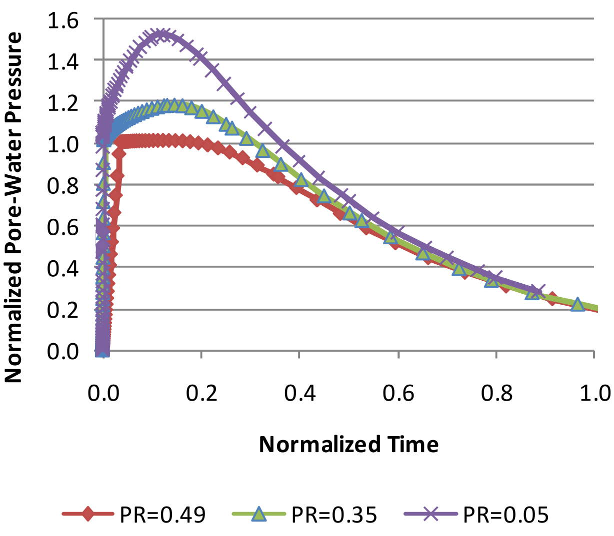

Figure 3 Demonstration of Mendel-Cryer effect with the SVSOLID / SVFLUX software

2.3 *1D Coupled Large-Strain Benchmarking*

The majority of previous large-strain consolidation formulations developed by Gibson and Schiffman for the phosphates industry took the form of 1D formulations developed in terms of void ratio (rather than stress-state variables). It is therefore important to demonstrate continuity with previous formulations as the current formulation is developed in terms of stress-state variables.

The "Scenario A" example as presented by Townsend (1990) is utilized for benchmarking purposes. In this example a 1D soil column at an initial void ratio of 14.8 is allowed to consolidate under its own self weight. The bottom boundary is a no-flow boundary and therefore flow of water can only be in the upward direction. The benchmark is extreme from the sense that the material may change from a void ratio of 15.0 to approximately 7.0 with a very small change in stress. The example is therefore highly non-linear and a challenging benchmark to solve. This benchmark was previously solved with a group of 1D academic codes which were largely based on the Gibson (1967) formulation. The geometry of the benchmark is shown in Figure 4.

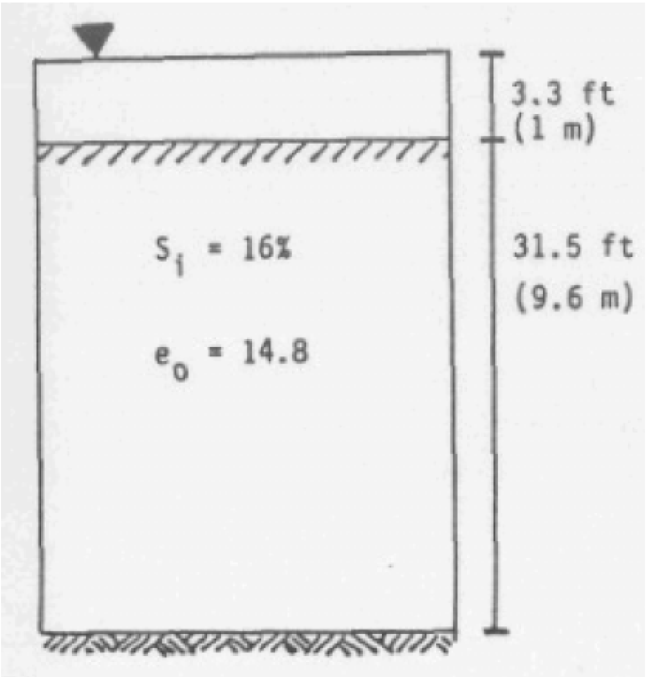

Figure 4 Townsend (1990) Scenario "A" benchmark

The results of the Townsend benchmark were compared to the SVSOLID / SVFLUX software package on three different aspects; i) height of the tailings, ii) pore-water pressure profile at the end of a 1-year period, and iii) void ratio profile at the end of a 1-year period. The results may be seen in Figure 5, Figure 6, and Figure 7. From these results it can be seen that there is reasonable comparison to existing Gibson-based 1D formulations. Small differences exist but may be attributed to unknowns which are not presented in the Townsend paper such as mesh resolution, time-step size, or the behavior of the upper boundary condition. For example, it is not mentioned in the paper if the upper boundary head deforms down during the consolidation process or is kept constant at its original height. In the SVSOLID / SVFLUX solution it is assumed that the upper head boundary condition is kept constant ov

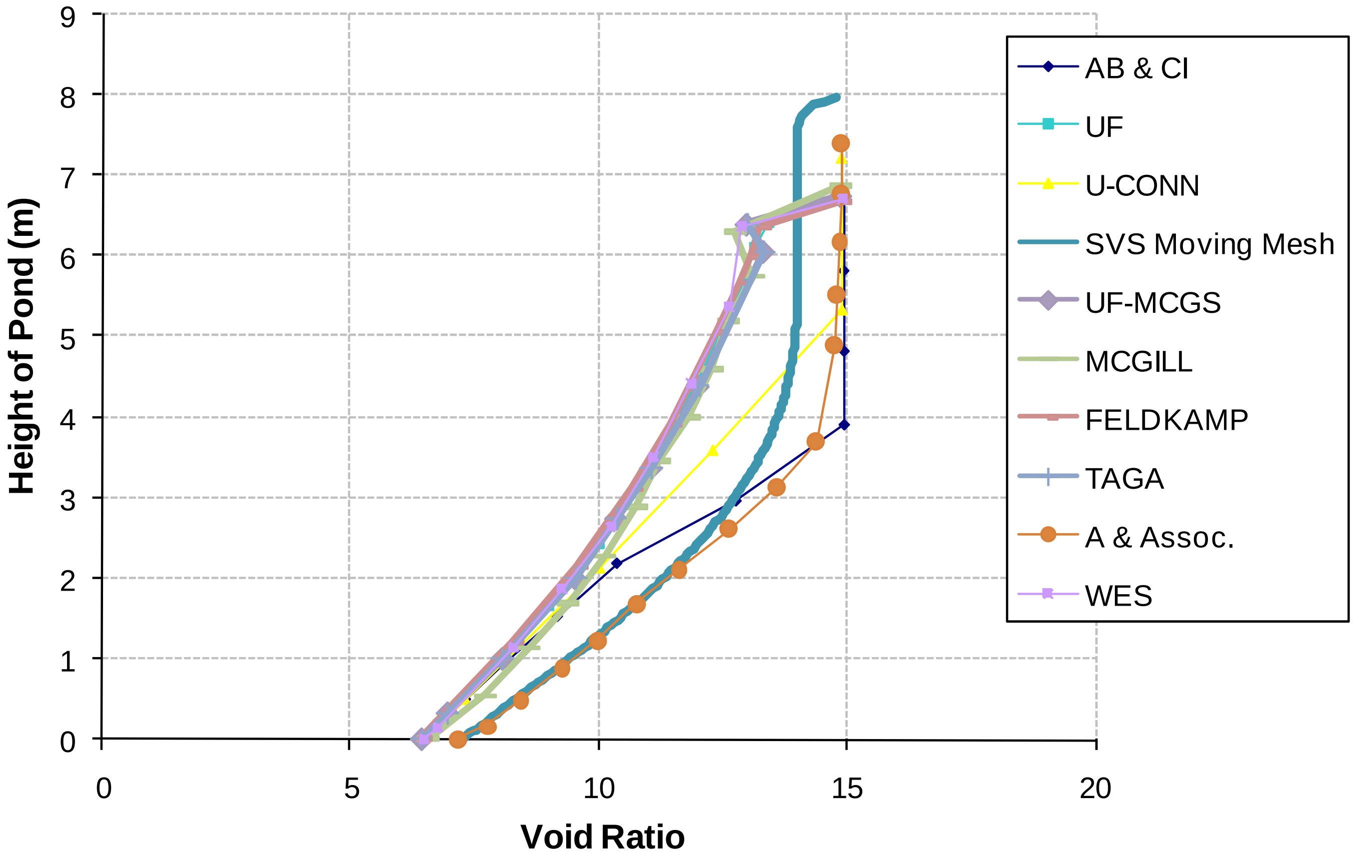

Figure 5 Comparisons to Townsend Scenario A for a void ratio profile after 1 year

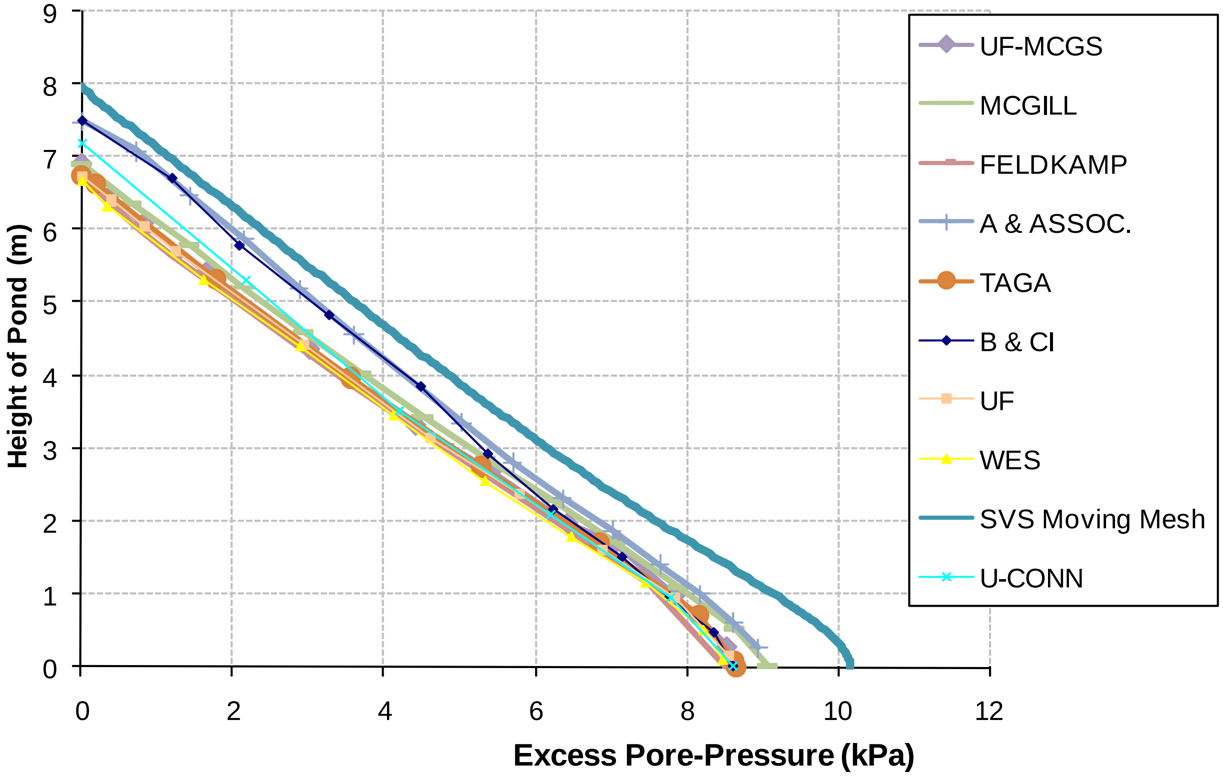

Figure 6 Comparisons to Townsend Scenario A for excess pore-water pressures after 1 year

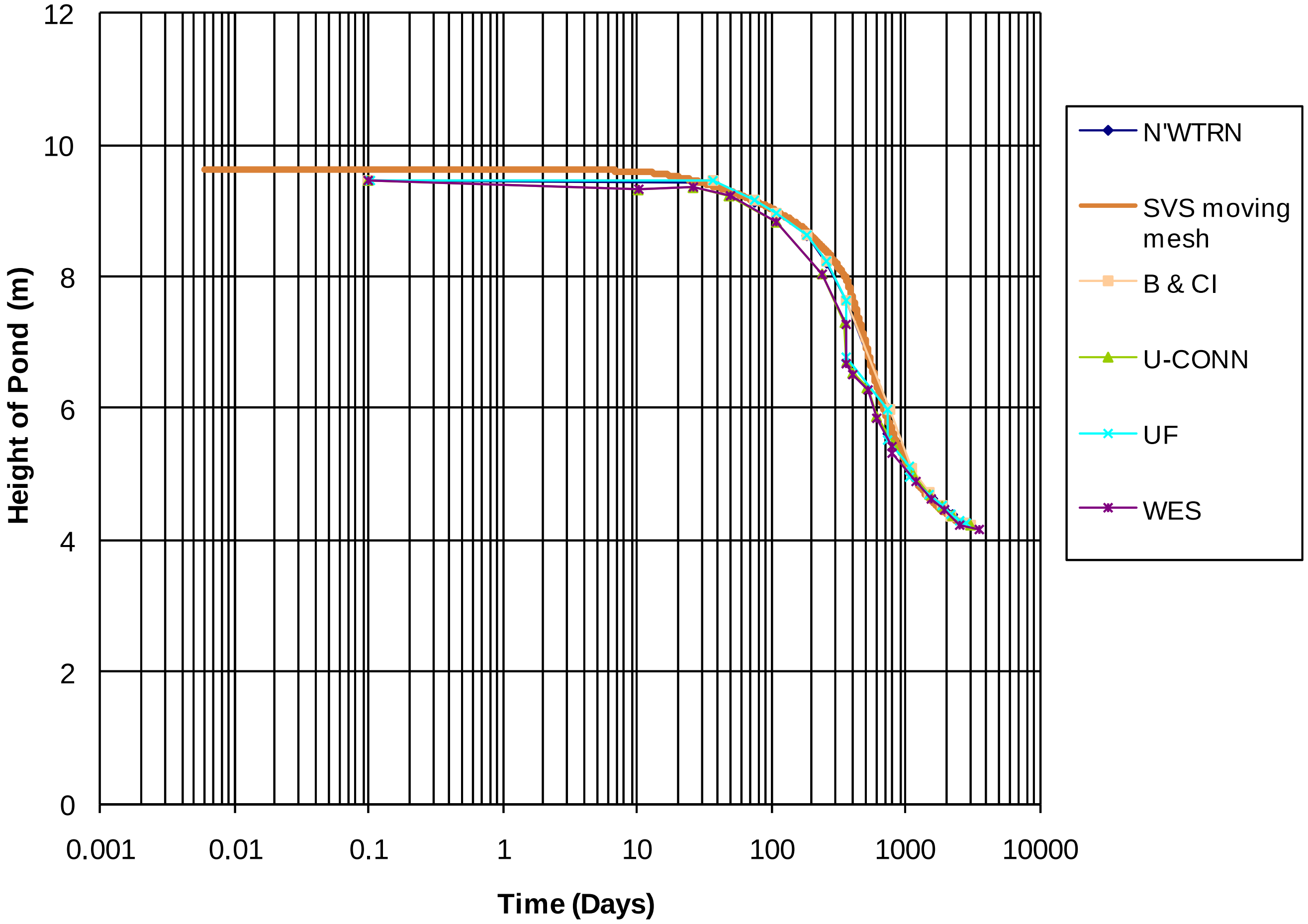

Figure 7 Height of surface of tailings versus time (Townsend, 1990)

3 WHY 2D AND 3D ANALYSIS?

Typical analysis of tailings areas for some mine sites has involved the use of a 1D large-strain numerical model. Such a model as a series of 1D profiles and the results may or may not be interpolated in some manner.

The primary difficulty with such an analysis is that it misses the fundamental processes of the influence of lateral strains on the solution outcome. It is therefore impossible to represent Mandel-Cryer type of effect using a 1D numerical model.

It is also possible for lateral drainage to occur in many TMAs. Drainage through lateral boundary conditions can only be truly represented in a 2D or 3D numerical model and directly influences the resulting profiles of excess pore-water pressures.

In summary, a 2D or a 3D analysis is warranted for the following reasons:

1. **Lateral flow:** In a 2D or a 3D model it is possible to track lateral flow. This may be of particular importance if the hydraulic conductivity of deposited layers is different in a lateral direction as opposed to a vertical direction.
2. **Side boundary conditions**: The amount of lateral drainage and its influence on pore-water pressures and void ratios within a TMA can only be determined with a 2D or 3D model. In many large pits the true drainage around the pit is irregular and is best modeled in 3D.
3. **Mandel-Cryer effect**: The Mandel-Cryer effect is only present in a 2D or 3D analysis and can result in a decreased dissipation of pore-water pressures due to the throttling of boundaries where drainage is occurring.
4. **Variable depositions**: Tailings may be deposited in a manner which yields defined zones of coarse, medium, and fine materials. If this is the case then zones can cause areas of differential settlement which are best modeled in 2D and 3D.

In summary a 2D or a 3D analysis is warranted in most situations because of the demonstrated influence of lateral strains. A calibrated 2D or 3D model can provide a significantly greater understanding of the fundamental processes involved over the lifetime of a TMA. Once the fundamental process is determined the ability to run multiple simulations and extend TMA lifetime is present.

4 LAYERED TAILINGS PIT ANALYSIS

The large-strain consolidation analysis is ideal for the evaluation of tailings when deposited in a pit. Such a storage facility is common for phosphate, copper, and uranium mine tailings. Tailings are typically deposited in a slurried form in somewhat of a continuous fashion. Therefore one of the issues is how to numerically model the continuous deposition of tailings. For the SVFLUX / SVSOLID large-strain implementation the deposition process can be replicated through the use of layers of materials which are phased in over a specific time period. For example, annual layers can be created and then applied to the numerical model at the start of each year. Each layer starts with specified initial properties and is allowed to consolidate due to its own self weight through the year. Such a phased / layered approach is consistent with previous recommended approaches (Gibson, 1958).

A typical setup of a series of layers is illustrated in Figure 8. In this particular model each year can be represented by an individual layer of material. The thickness of each layer is determined by the volume of slurried tailings deposited in a particular year. Boundary conditions of this numerical model can be represented by a variety of standard load / fixed / free / head / flow boundary conditions as relevant to the individual flow and stress distribution components of the analysis.

Average material properties were input for the current analysis with void ratio as a function of net normal stress and hydraulic conductivity as a function of void ratio.

The model at various times may be seen in the series of following figures. The flow out of any side of the numerical model can be tracked to determine reasonable discharges to the environment. The upper flow boundary can be represented as a head or flux boundary conditions which could be used to represented tailings deposited subaqueously or exposed to the atmosphere.

Once the model is solved the results can be viewed in terms of total deformations at any time, height of tailings with time, and pore-water pressure and void ratio profiles vs. time and depth.

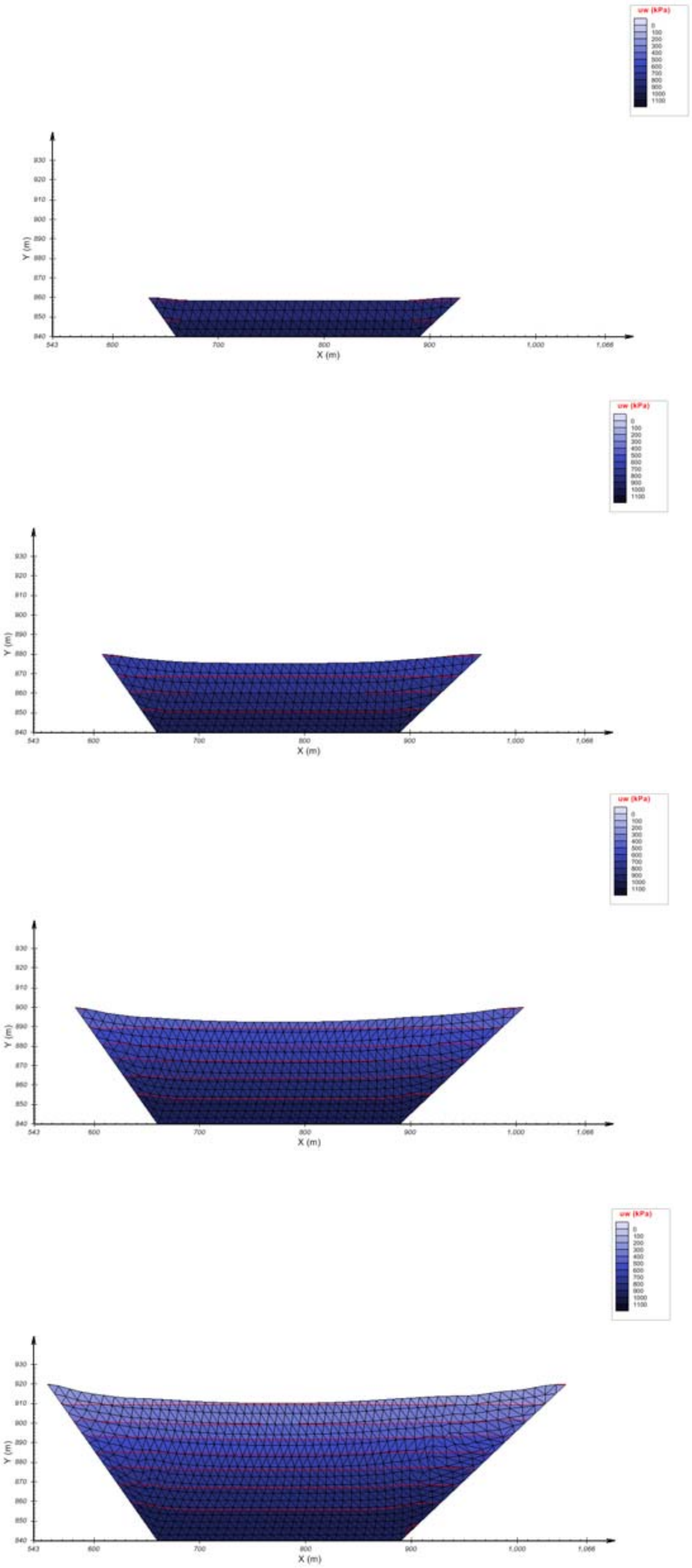

Figure 8 Example analysis of a tailings pit implementing annual layers

5 CONCLUSIONS / DISCUSSION

It can be seen from the results of the benchmark models that the new SVSOLID / SVFLUX software can successfully replicate published results for individual large-strain examples as well as fully coupled examples. The code duplicates previous implementations of the 1D large-strain formulation and then extends these same formulations to 2D and 3D.

The use of this formulation is ideal for the estimation of the long-term performance of mine tailings. The use of 1D theory only will compromise the analysis such that pore-water pressures would be under-estimated and tailings consolidation times would be under-estimated. A 2D or 3D analysis will allow inclusion of the Mandel-Cryer effects and allow improved representation of the underlying physical processes. Once the underlying physical processes have been understood the model can aid in improved storage capacity for TMAs.

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