Moving from 2D to a 3D Unsaturated Slope Stability Analysis

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ABSTRACT

Two-dimensional (2D) limit equilibrium analyses remain the most common method of analysis in slope engineering practice. It is commonly generally perceived that 2D slope stability analysis always provides a more conservative estimate of the 3D slope stability problem. Most previous studies comparing 2D and 3D stability analysis also ignore the effect of negative pore-water pressures (i.e., matric suctions) in the soil zone above the groundwater table. In this paper, a comparison study is reported between 2D and 3D slope stability analysis for soil slopes with a portion of the soil profile having matric suctions. The differences between a 2D and a 3D factor of safety are found to be heightened when unsaturated conditions are considered. The paper also presents a framework for the inclusion of soil suctions in the calculated factor of safety for geotechnical engineering practice.

A discussion on reducing a 3-D numerical model to give answers similar to a 2-D slope stability analysis will be presented. A basis for transitioning to 3-D analysis will be developed.

INTRODUCTION

The traditional application of the limit equilibrium method (LEM) has been in the context of a 2-D plane strain analysis. Geotechnical engineers have become complacent with the use of 2-D slope stability as it is easy to perform. It should be noted that 2-D analysis suffers from fundamental limitations foremost of which i) the slip shape is assumed to be cylindrical, and ii) the slope geometry is assumed to be unchanged in the 3rd dimension, and iii) the geo-strata is assumed to be unchanged in the 3rd dimension. Such assumptions have proven 2-D analysis to be conservative with respect to the true 3-D factor of safety in the amounts between 10-50% (Domingos, 2016). If unsaturated aspects of slope stability are considered then the difference can be as high as 60% (Zhang, 2015). Therefore significant opportunity exists to optimize existing designs through the application of 3-D slope stability analysis. It should be noted that the difference between a 2-D and a calculated 3-D factor of safety is different in each situation. Therefore it is impossible to assume a specific 2-D/3-D difference for a particular scenario.

The theory for the 3-D LEM has been in existence for decades and is therefore not new. Early theoretical development efforts are available in research literature (Howland, 1977; Hungr, 1987, Hungr et al., 1989, and Fredlund, 1977).

The application of 3-D slope stability LEM in practical geotechnical analysis requires consideration of the effects of i) **slip surface shape** being elliptical, block-based, or composite, ii) the effects of **ground surface geometry,** and iii) the effects of **geo-strata varying**. The effects of **material constitutive behavior** (such as unsaturated shear strength) may also cause additional variations in going to a 3D analysis.. This paper will focus on the effects of slip surface shape, ground surface shape, and the influence of soil suctions on the computed factor of safety. A recommended approach for the application of 3-D slope stability analysis in the practice of unsaturated geotechnical engineering will also be presented.

The application of software tools to analyze slope stability in 3-D has traditionally been highly limited. Recent developments allow for the easy application of 3-D stability analysis in typical problems (Fredlund, 2015; Reyes, 2014). The SVSLOPE software developed by SoilVision Systems Ltd. is utilized for the analysis presented in this paper.

CONTINUITY BETWEEN 2D AND 3D LEM

It is of importance to understand the difference between the slip shape analyzed in a 2-D analysis as opposed to a 3-D analysis. In a 2-D analysis the slip shape is ultimately extended infinitely in the 3rd dimension and shear on the end surfaces is not considered. An example 2-D analysis is presented in Figure 1. The equivalent 3-D analysis is shown in Figure 2 where there is no shear strength applied to the vertical end surfaces.

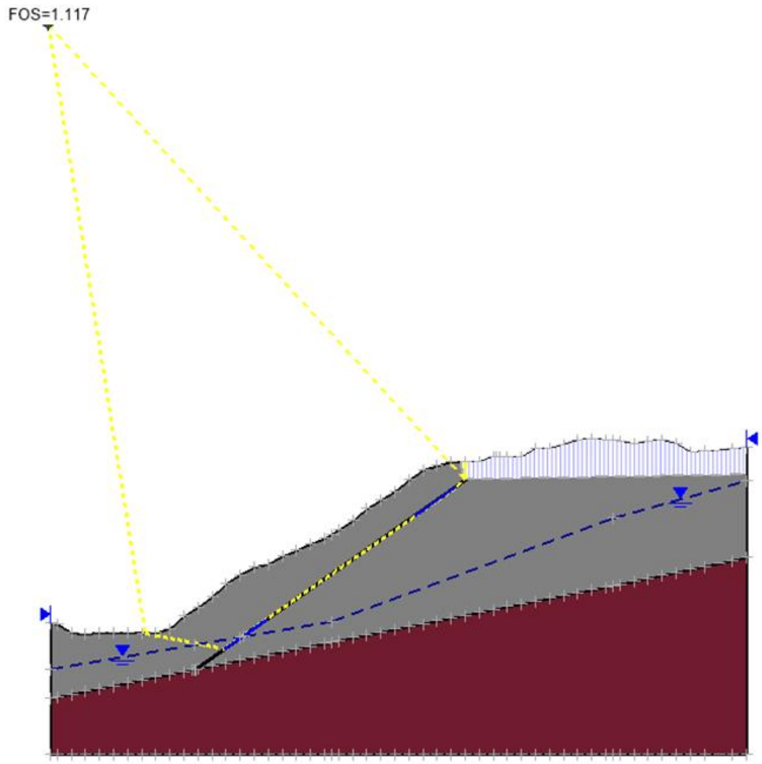

Figure 1 Example 2-D stability model

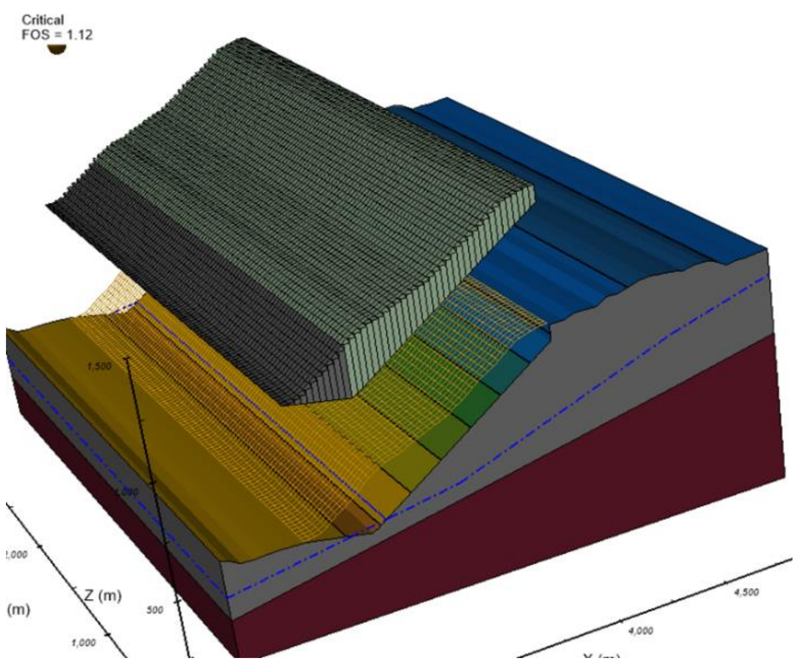

Figure 2 Equivalent 3-D analysis (slip surface exploded out of slope)

It should be noted that a simple proof of a 3-D analysis can be done by creating a 3-D model of a 2-D extruded slip surface and applying zero shear strength to the end-walls. This is an easy way to prove the 3-D equivalent scenario of any 2-D analysis. It also highlights the fundamental limitation of a 2-D analysis which i) considers the slip to be of infinite length in the 3rd dimension and ii) does not consider the influence of shear strength on the end surfaces.

SLIP SURFACE SHAPE EFFECT

If an engineer wants to strictly determine the difference that slip surface shape makes then it is easiest to extrude a 2-D profile model out to 3-D space to determine the factor of safety of a slip surface assuming an ellipsoidal slip surface. It is recognized that not all 3-D slip surfaces are of a perfect ellipsoidal shape but a majority of slips are an ellipsoidal shape. Therefore it was considered a reasonable starting assumption by the authors.

The benefit of this type of analysis is it allows the engineer to determine the effect on the factor of safety (FOS) of moving from a 2-D plane-strain analysis to a 3-D analysis with an ellipsoidal slip surface. When the slope is extruded to 3-D then an aspect ratio for the ellipsoid must be found through a trial-and-error process. For the present paper a number of example models were selected as benchmark 2-D models and a few are presented in **Error! Reference source not found.** and **Error! Reference source not found.**. These models were selected out of list of classic slope stability benchmarks as compiled by SoilVision Systems Ltd.

A common question related to 3-D analysis is: "Won't a 3-D analysis of an extruded profile tend to an ever wider and wider 3-D elliptical slip surface as it tries to approach the 2-D result (with its lower FOS)?". One of the key aspects to answering this question is related to the assumption of the slip surface shape. If the shape is cylindrical with small beveled or rounded ends, then as the cylinder becomes longer and longer the effects of the shear on the ends theoretically becomes a smaller and smaller percentage contribution. It should be noted, however that the "extended cylinder" described in the above argument is not a common slip surface failure shape in the field. If we assume an elliptical slip surface failure, then the results illustrate that the failure does not tend to be infinitely wide as previously assumed.

Figure 3 shows the results of extruding the VS_24 model to an equivalent 3-D numerical model. It can be seen from this extruded model that a wide range of aspect ratios for the ellipsoid have been tried (the trial slip surfaces have been plotted) and the ellipsoid with the minimum FOS corresponds to an aspect ratio of 16.8 with a minimum FOS of 1.72 which is a 19.6% increase over the 2-D FOS of 1.438. The results of extruding three such 2-D models to 3-D are shown in Table 1. It can also be seen from these results that the difference for simple and low-angle slopes between the 2-D and 3-D FOS ranges between 4.3% to 20%.

Table 1 Results of 2-D profiles extruded to 3-D

	2-D		3-D			
Model	FOS	Max Depth (m)	FOS	Aspect	Max Depth (m)	Diff
VS_1	0.988		1.031	4.8	3.013	4.35%
VS_3	1.375	4.76	1.45	8.6	4.56	5.45%
VS_24	1.438	6.889	1.72	16.8	6.814	19.61%

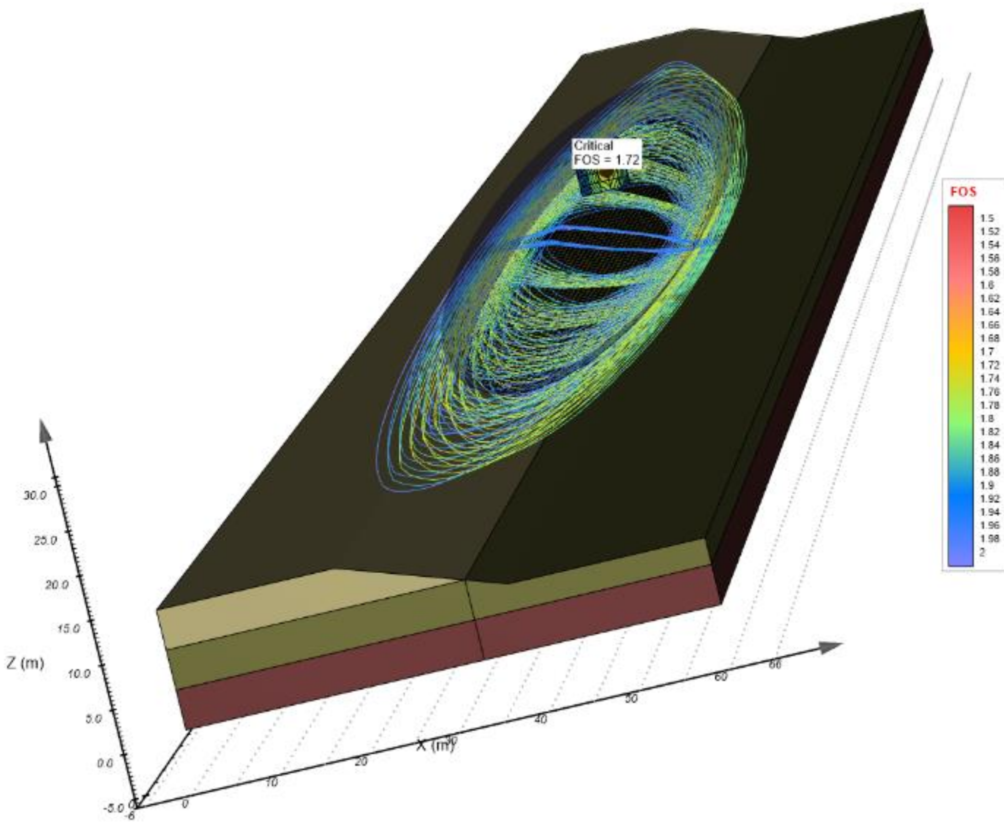

Figure 3 VS_24 Extruded 3-D model with critical slip surface

GROUND SURFACE TOPOGRAPHY VARIATIONS

The second primary reason why 3-D analysis differs from 2-D analysis is that there may be variation of the surface topology Such variation can produce differences in the computed FOS. Such variations can largely be grouped into surface shapes which are convex or concave. While it is recognized that typical topography may be highly variable, the general trends can be easily quantified with simple concave and convex shapes. Such an analysis has been performed (Zhang, 2015; Domingos, 2014; Adriano et al., 2008) and the geometry for the comparison can be seen in Figure 4.

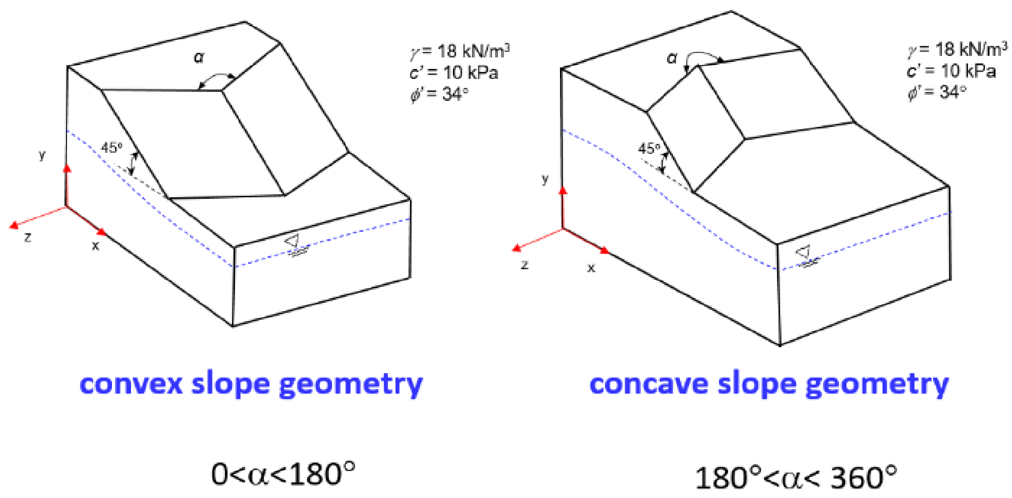

Figure 4 Convex and concave geometries used in 3-D simulations

From analyzing the geometry it can be seen that i) the consideration of such geometries results in an increase in the computed FOS over a 2-D scenario and ii) the consideration of unsaturated shear strengths further increases the computed FOS in 3-D (Figure 5). It can also be surmised that the consideration of 3-D geometries in a 3-D numerical model can often result in computed FOS values which are higher than 2-D analysis and have the potential to save on construction costs in the design of slopes.

A simple analysis may be performed to determine the potential differences between 2D and 3D analysis as well as between a saturated and unsaturated analysis. The factors of safety on simple geometry slopes with a steep angle and a low slope angle are investigated under various shear strength parameters and groundwater conditions. For simple slopes with a low slope angle, the difference in factor of safety between a 2D and 3D slope stability analysis, (i.e., $\Delta Fs/Fs_{2D}$), monotonically increases with an increase in c', ϕ', and ϕ^b values. The variation in the factor of safety (i.e., $\Delta Fs/Fs_{2D}$), of the low angle, simple slopes with various combinations of c' and ϕ' values ranges from 4% to 9% with ϕ^b equal to zero. When ϕ^b is assumed to be 15 degrees, the values of $\Delta Fs/Fs_{2D}$ range from 9 to 16%. It is found that the difference of factor of safety between 2D and 3D analysis for a steep, simple slope is generally larger than that of a low angle, simple slope. When ϕ^b is 15 degrees, the values of $\Delta Fs/Fs_{2D}$ for the simple, steep slope range from 12 to 18% with various combinations of c' and ϕ' values. However, for a simple, steep slope, $\Delta Fs/Fs_{2D}$ does not increased monotonically with the shear strength parameters (i.e., c', ϕ', and ϕ^b). The effect of groundwater table on the difference between 2D and 3D factors of safety is more pronounced for the simple, steep slope than for the low angle slope. These differences are summarized in Figure 5.

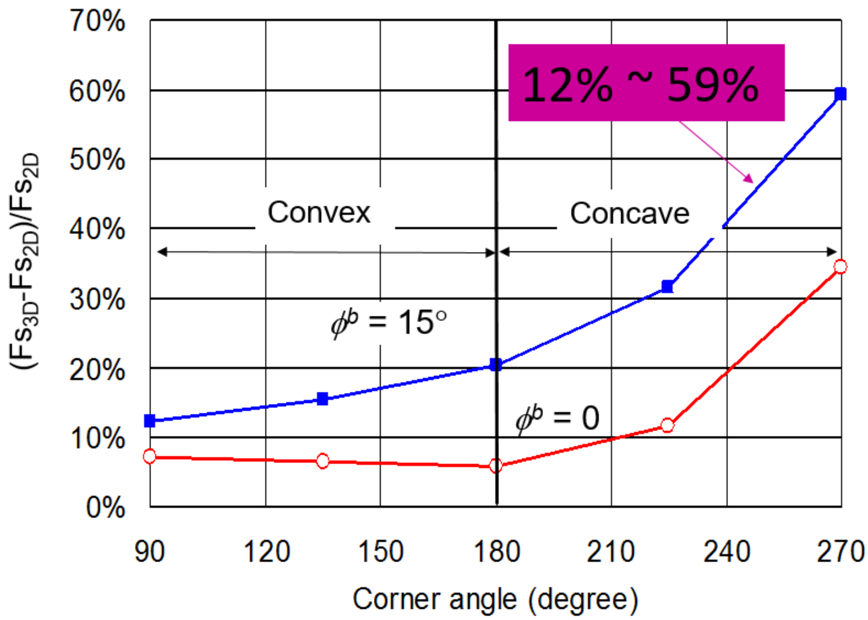

Figure 5 Differences between 2-D and 3-D computed FOS for convex and concave saturated/unsaturated slopes (Zhang, 2014)

The results of the influence of surface topology suggest that the consideration of such influence may not be trivial. The consideration of 3-D effects of topology may therefore be useful in the practical application of 3-D slope stability.

PRACTICAL APPLICATIONS GUIDELINES

The current state of practice in geotechnical engineering has a long history of application and design of earth structures. It must be recognized that the existing limits of design of 1.3 and 1.5 have been developed in the context of an application of a 2-D limit equilibrium numerical model. Common questions related to the application of a 3-D numerical analysis to slope design are “Can I use a 3-D analysis and design for my same limits of 1.3 and 1.5?”, or “Should we move to only use a 3-D analysis?”. There is recognition that significant further work is required to completely understand a 3-D analysis, but principles can be developed which form a foundation for a path forward in terms of a practical application of a 3-D numerical model in geotechnical engineering practice.

Firstly, it is recommended that geotechnical engineers begin to run both 2-D and 3-D numerical models to determine the potential differences between such analysis. There is not a single percentage that represents a consistent difference between a 2-D and a 3-D analysis and therefore the geotechnical engineer must begin to become familiar with the results of the analysis in each engineering situation.

It is important to understand the logic followed by existing analysis performed over the past few decades. The process may be illustrated in Figure 6. From this figure we may note that i) the definition of failure at a FOS=1.0 does not change between 2-D and 3-D analysis, ii) A 3-D FOS will typically be higher than a 2-D FOS, and iii) when a 3-D analysis is performed we are moving closer to field conditions and therefore moving closer to the real-world FOS.

It should also be understood that the true FOS is really comprised of a KNOWN and an UNKNOWN FOS. When a 2-D analysis yields a value of 1.3, the known FOS_{2D} is 1.3 or 30%. The unknown FOS_{2D-3D} is the difference between the 3-D and the 2-D FOS. The geotechnical engineer traditionally has not known what the second FOS_{2D-3D} is but they have a certain degree of comfort knowing it exists. However, the second unknown FOS_{2D-3D} can be a difference between 10-60%. On both ends of this range there requires further consideration. If the FOS_{3D} is 10% over the FOS_{2D} then it must be realized that there is little "cushion" over the known FOS_{2D}. If the FOS_{2D-3D} is closer to 60% or higher then there is significant overdesign that may be costly.

When a 3-D slope stability analysis is performed the engineer, they are merely determining the unknown FOS_{2D-3D} is that has existed but has remained previously unknown. This additional piece of information gives the geotechnical engineer another crucial piece of information required for slope stability analysis.

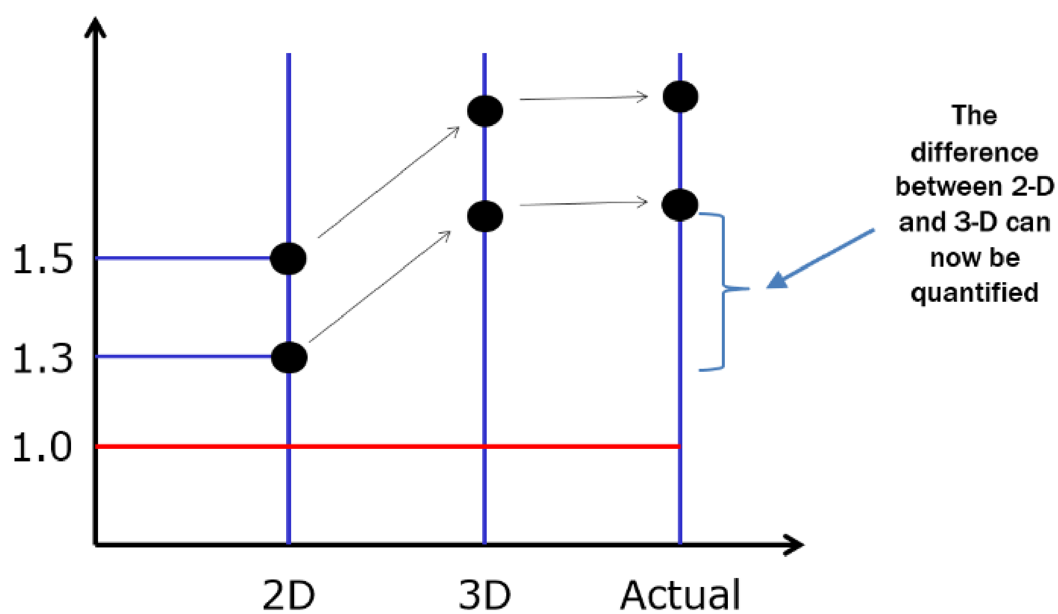

Figure 6 Illustration of 2-D/3-D analysis

A common question is "Can the engineer use a 3-D analysis along with the existing factors of safety of 1.3 and 1.5?". It should be noted that the existing limits were developed based on the assumption for an additional and unknown conservatism. If a 3D analysis yields a FOS=1.3 the geotechnical engineer must realize that they truly only have a 30% overdesign with no additional conservatism built in. It is suggested that geotechnical engineers will have to run both 2-D and 3-D analysis over the coming years to ensure that they become familiar with the true FOS limits which create sufficient comfort for designs.

Approaching 3-D analysis in steps may be an appropriate manner in which to educate the client about the 3-D influences of the FOS calculation. As illustrated in this paper the engineer can first i) run a 2-D analysis, ii) extrude the model to perform a simple 3-D analysis, and iii) then run a full 3-D analysis considering all 3-D geometry. This will yield a progression that will educate both the geotechnical engineer and their client on the additional FOS contributed from representing the slip surface more accurately as well as from representing the topology and the geo-strata more accurately in the 3rd dimension.

CONCLUSION

As 3-D analysis techniques become more common and easily applied the geotechnical engineer must understand the differences between a 2-D and a 3-D analysis. The primary differences in the 3-D slope stability analysis lie in the fact that i) the slip shape is more realistic, ii) the topology may be varying in the 3rd dimension, iii) the geo-strata may vary, and iv) the material constitutive model may vary spatially.

This paper has illustrated how it is easy to extrude a numerical model and determine the influence of the improved slip shape easily. It can also be seen that it is straightforward to incur FOS_{3D} which are 4-20% higher than their 2-D plane strain equivalent models in this process. Such an analysis can identify the specific influence of the slip surface shape on the FOS calculations.

The influence of surface geometry on the calculated FOS can also be illustrated through the representation of typical concave or convex geometries. Such an analysis was performed by Zhang (2014) and illustrated differences between 2-D and 3-D FOS values of between 8-60%. A higher difference was incurred if unsaturated soil mechanics were considered.

This paper presents a step-wise methodology for the application of 3-D slope stability analysis in the practice of geotechnical engineering. The paper also presents some foundational principles for the understanding of how a 3-D analysis fits in the existing design limits of 1.3 and 1.5.

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